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MASTER

TITLE: EARLY HISTORY OF PHYSICS WITH ACCELERATORS

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———— DISCLAIMER ————

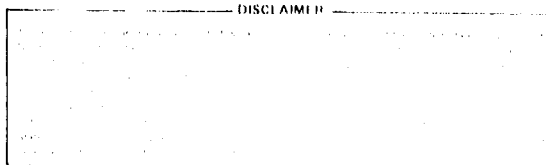# Los Alamos
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Early History of Physics with Accelerators

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Abstract. - The early history of physics at accelerators is reviewed, with emphasis on three experiments which have had a profound influence on our view of the structure of matter: The Franck and Hertz experiment demonstrating the mechanism of atomic spectra, the Cockcroft and Walton experiment opening practical ways of studying nuclear disintegration, and the discovery of the ∇^{++} isobar of the proton by Fermi and collaborators, revealing structure in the nucleon. Fermi's work is illustrated by pages from his notebooks.

Resume. - L'histoire du debut de la physique des accelerateurs est presentee, avec l'accent sur trois experiences qui ont eu une influence profonde sur notre conception de la structure de la matiere: l'experience de Franck et Hertz quie demontra le mecanisme des spectres atomiques, l'experience de Cockcroft et Walton quie ouvrit des voies pratiques a l'etude de le desintegration nucleaire, et la decouverte de l'isobere ∇^{++} du proton per Fermi et ses collaborateurs, quie revela une structure dans le nucleon. Le travail de Fermi est illustre par des extraits de son journal de laboratoire

Introduction

I don't intend to give a comprehensive survey of all the important experiments in elementary particle physics that were carried out at accelerators in the period 1930 - 1960. Instead, I'd like to tell about three accelerator experiments that in a dramatic way, changed physics profoundly, each in its own time. The experiments I have in mind are the following:

1) "Excitation of the the 2536 Å Resonance Line of Mercury," J. Franck and G. Hertz, (1914).[1,2]

2) "Disintergration of Elements by High Velocity Protons," J. D. Cockcroft and E. T. S. Walton (1932).[3,4]

3) "Total Cross-sections of Positive Pions in Hydrogen," H. L. Anderson, E. Fermi, E. A. Long, and D. E. Nagle. (1952).[5]

The first experiment made it clear that Bohr's theory was correct and thereby opened the way to a proper understanding of atomic spectroscopy. The second opened the field of nuclear spectroscopy. The third, by making evident the significance of isotopic spin and revealing the existence of an excited state of the proton, provided the key to the 3rd spectroscopy, the spectroscopy of the hadrons.

The idea that each successive stage in the development of elementary particle physics was marked by a new spectroscopy is taken from Weisskopf. In an article,[6] "What is an Elementary Particle," written in celebration of the 50th anniversary of the International Union of Pure and Applied Science, he discusses how structure in an elementary system is always revealed by a spectroscopy. He identified three stages, and wrote, "It is historically interesting that these three progressive steps toward a deeper understanding of the fundamental structure of matter were

initiated by discoveries made almost exactly 20 years apart: the discovery of the nuclear atom by Rutherford in 1911, the discovery of the neutron by Chadwick in 1932, and the discovery of the excited Δ-state of the proton by Fermi and collaborators and its interpretation by Brueckner and Watson in 1952." When he wrote this article in 1972 a 4th spectroscopy of quarks and gluons, that occupies a large part of high energy physics today, was emerging.

Franck and Hertz Experiment

The Rutherford scattering experiment gave no suggestion of a spectroscopy until Bohr's theory provided it. The experimental demonstration that atomic spectroscopy could be understood from the point of view of Bohr's theory was made by Franck and Hertz. This was not the classic Franck and Hertz experiment in which it was shown that an electron would lose 4.9 volts, and not less, in inelastic collisions with mercury atoms. It was the one that followed and answered the question "What happened to the lost energy.?"[1] Fig. 1 shows the apparatus. It is an accelerator small enough to be held in one hand. There is a platinum filament, labeled D in the figure, that emitted electrons when heated by an electric current. The electrons were accelerated toward the anode N when this was held at positive potential with respect to the filament. The bulb was filled with mercury vapor that served as the target. With anode voltages in excess of 4.9 volts, inelastic collisions between electrons and mercury atoms took place within the bulb. To see what came out of this, an ultraviolet spectrograph was set up to analyze any possible light emission. The spectrogram obtained is reproduced in Fig. 2. The result is shown in the lower spectrum. The darkened continuous region on the right is due to the light emitted by the hot filament. Off to the left there is a single
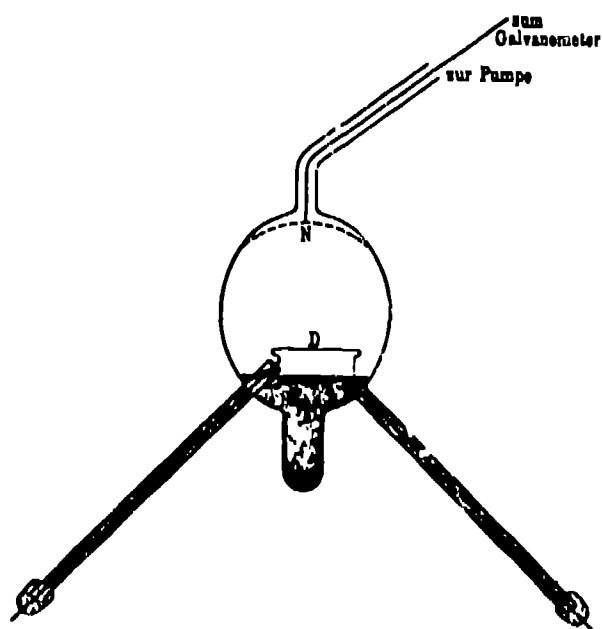

Fig. 1: The electron accelerator of Franck and Hertz used to excite the 2536 Å resonance line of mercury.
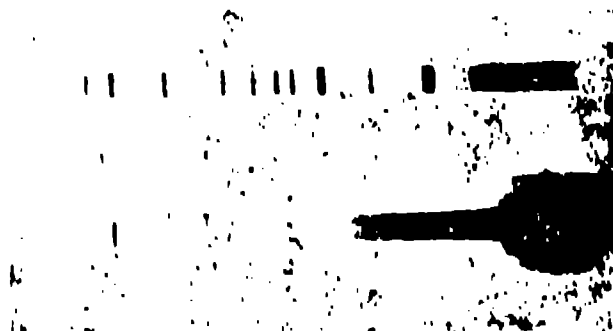

Fig. 2: Ultraviolet spectrogram showing the single 2537 Å resonance line of mercury (below) and the comparison spectrum (above).

isolated dark line, identified by the comparison spectrum of mercury above, as the 2536 Å resonance line of mercury. We recognize this line today as coming from the first excited state of the mercury atom and we know that it arises by the emission of radiation following excitation by electron collision. In fact, from their measurement of the electron energy, 4.9 volts, and the wavelength of the emitted light, Planck's constant was determined. The value they obtained, h = 6.59 x 10^{-27} ergs , agrees, within errors, with the present value, 6.63 x 10^{-27} erg sec.

If this seem obvious enough now it is because we know Bohr's theory. But it may interest you to know that when Franck and Hertz did their experiment they didn't know about Bohr's theory. It had been published some six months earlier, but they hadn't heard of it. They were negligent not to have read about it in the literature. You know how that happens. There was an active seminar in Berlin at the time at which all the latest developments in physics were discussed. But if Bohr's theory had been presented there it wouldn't have been taken seriously. In fact, in a letter to Bohr, Richard Courant once wrote, ".....how glad I was when I read of the Nobel Prize report in the newspapers. It reminded me vividly of that beautiful day in Cambridge in 1913 when you set forth your ideas for me in the quadrangle of Trinity. Thanks to prior suggestion by Harald (Bohr), who had so often told me wonderful things about his brother, I was at that point immediately ready to believe that you might be right. But when I then reported of these things here in Göttingen, they laughed at me that I should take such fantasies seriously." However, the agreement with Bohr's ideas was so striking that no one could deny their correctness. There followed a rapid development in the theory of atomic spectra and a revolution in the understanding of the nature of the atom. When we think how much modern man depends on the chemistry, the biology, and the technology that grew out of the secure knowledge of atomic structure, we begin to have a measure of the power and the importance of those developments.

Cockcroft and Walton Experiment

While the discovery of the neutron was essential to the understanding of the nucleus, it was the Cockcroft-Walton accelerator and the experiments they did with it that opened the field of nuclear spectroscopy. Both experiments were done in 1932. Already in 1919, Rutherford had shown that the nucleus could be disintegrated by alpha particles.[8] However, his alpha particles were those emitted from naturally occurring radioactive elements. There were not enough of them to carry out an extensive study of the phenomenon. The advances in electrical technology in the years following World War I made it possible to contemplate the production of high speed particles by artificial means. In 1927 Rutherford, as President of the Royal Society, expressed the wish for a supply of "atoms and electrons that have an individual energy far transcending that of the particles from radioactive bodies." To overcome the Coulomb barrier of the nucleus it would be necessary to have particles accelerated to energies of several million volts or more. This became the goal of those who contemplated building such machines. In fact, by 1932, Lawrence and Livingston[9,10] at Berkeley had constructed a cyclotron that accelerated protons to an energy exceeding 1 million volts.

Some years earlier, Gamow[11] and also Condon and Gurney[12] showed that wave mechanics explained how alpha particles could escape from the nucleus with an energy far below the Coulomb potential barrier. When Gamow was visiting the Cavendish Laboratory in 1928, Cockcroft inquired about the inverse problem -- the energy that would be required for a proton to penetrate the nucleus of a light element. The same principle applied and Cockcroft prepared a memorandum for Rutherford showing that there was a high probability for the boron nucleus to be penetrated by a proton of only 300 kilovolts energy. The conditions for lithium were even more favorable. Rutherford then agreed that work on this project could begin. The result was a d.c. accelerator based on the voltage doubler principle capable of developing 600 kilovolts.[4] Figure 3 is a photograph of the Cockcroft-Walton accelerator showing John Cockcroft sitting inside the small observation box in the foreground.
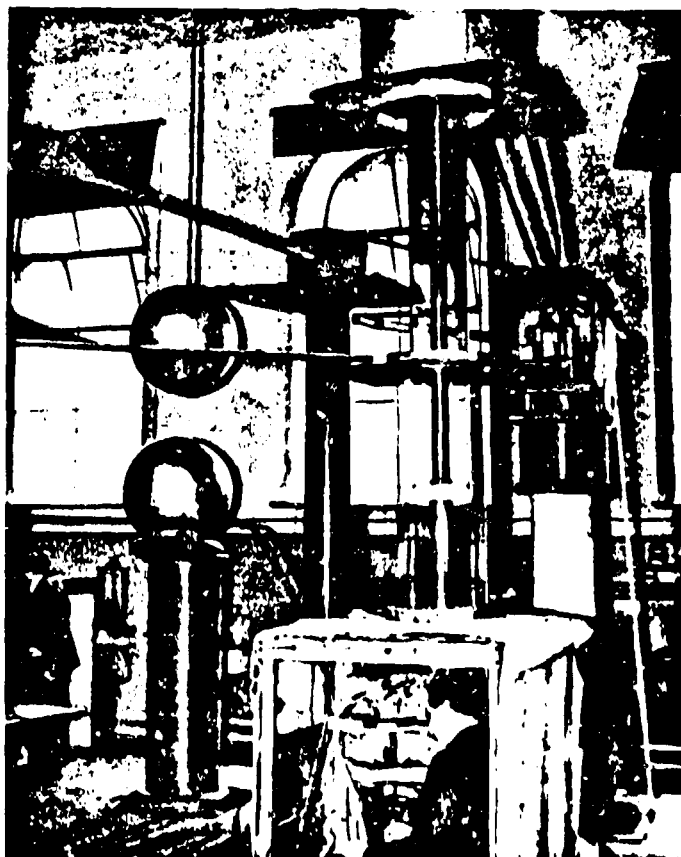
Fig. 3: The Cockcroft-Walton accelerator. John Cockcroft is sitting inside the small observer's box in the foreground.

The disintegration of lithium by protons was demonstrated by Cockcroft and Walton with an energy of only 125 kilovolts. The apparatus they used is shown in Fig. 4. The beam of fast protons was directed against a lithium target and the alpha particles from the reaction $p + Li \rightarrow \alpha + \alpha$ were detected by the well tried tool of Rutherford, the zinc sulphide screen. They then confirmed the reaction by demonstrating that the alpha particles were emitted in pairs. They used a primitive form of coincidence experiment, carried out with two zinc sulphide screens and two observers tapping keys. The resolving time was a second or so, somewhat longer by a factor of 10^8 than the resolving time of modern coincidence circuits. Disintegrations under proton bombardment were seen also for many other elements, not only with the zinc sulphide screen, but with other detectors that were available in the laboratory: the ionization chamber, linear amplifier and oscillograph of the type described by Wynn-Williams and Ward, and the Shimizu expansion chamber.

The disintegration of lithium might have been seen at Berkeley before it had at Cambridge, but the planning of physics experiments did not parallel the construction of the machines that were needed to perform them. Artificial radioactivity and fission could also have been discovered first at Berkeley if the focus and the tradition had been more on the physics than on the machines. Nevertheless, the Berkeley cyclotrons were widely copied and had a profound influence on the development of nuclear physics all over the world.[10]

Accelerator Development

Figure 5 shows Livingston and Lawrence standing inside the yoke of the magnet for the 37-in cyclotron. The magnet was one of a pair that had been built by the Federal Telegraph Company for a type of radio transmitter, the Poulsen arc generator made obsolete by the vacuum tube. My own introduction to cyclotrons came through John R. Dunning whose assistant I became. At Columbia University he managed to

Stream of fast protons
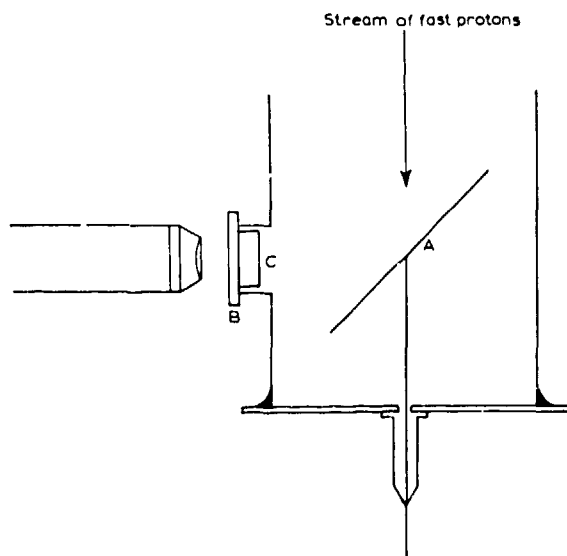
Fig. 2.

Fig. 4: Apparatus for detect-
ing the distintegration of
lithium by protons using a zinc
sulphide screen as detector.
Coincidence apparatus for p+Li
→ α+α using two ZnS screens.

Stream of
fast protons

ZnS   ZnS

B   A

Thin lithium
target
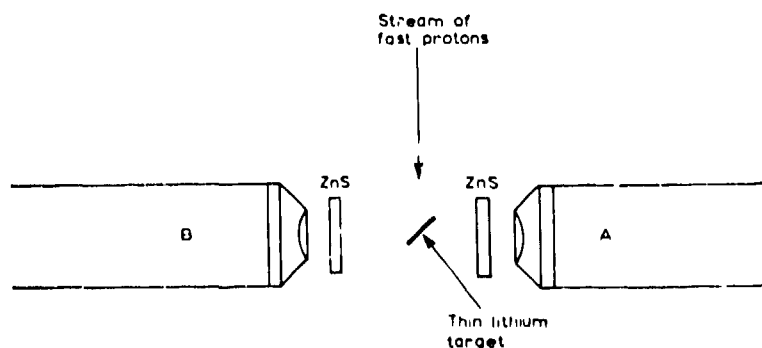
Fig. 3.
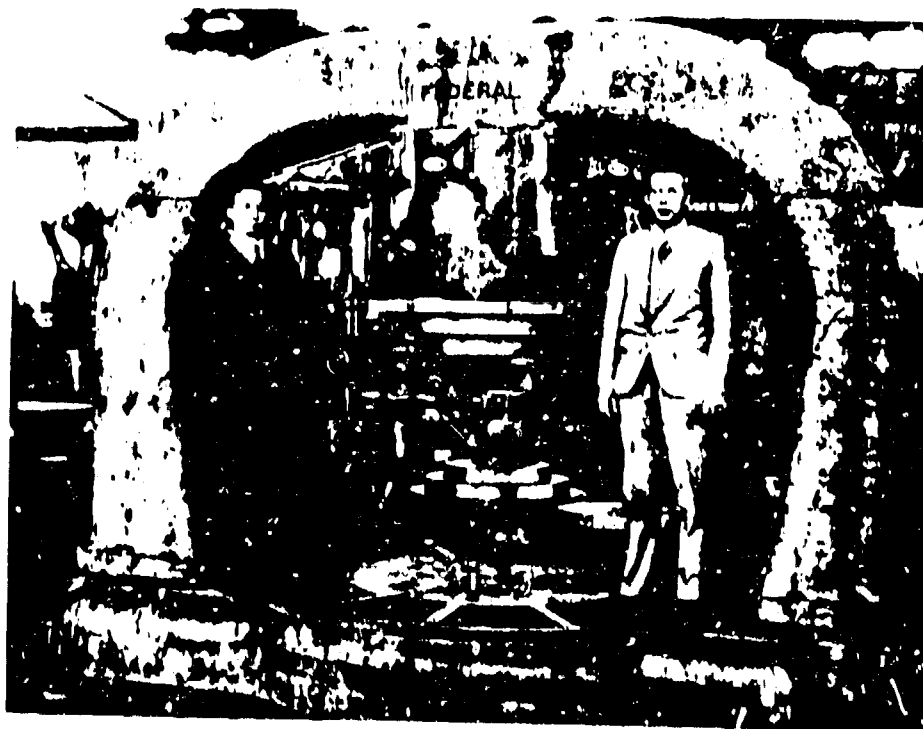
Fig. 5: Livingston (left) and Lawrence (right) standing in the yoke of the
magnet for the 37" cyclotron. It operated initially as a 27" cyclotron in
December 1932, and produced 4.8 MeV hydrogen ions.

fission of uranium, following the discovery of that phenomenon at the beginning of that year. Figure 6 is a photograph of the Columbia cyclotron that shows me carrying out an experiment on the resonant absorption of neutrons by uranium.

Figure 7 reproduces a graph taken from a report prepared by W. K. F. Panofsky.[13] It shows how the energy of accelerators developed over the years. The particle energy, either electron or proton, as the case might be, increased tenfold every six years over the 50 year period from 1932 - 1982. For the purposes of the graph, the energy plotted is the laboratory energy of the particles accelerated. For colliders, an equivalent energy is plotted which is the laboratory energy on a fixed target with the same center of mass energy. The plot shows how, as each technology began to reach its limit in energy, a new higher energy technology was invented to succeed it.

In 1960, the cut-off date for this colloquium, the 30 GeV proton synchrotrons at CERN and Brookhaven were just coming onstream, but the 6 GeV Bevatron at Berkeley had been in operation for several years. With it came the discovery of the anti-proton and a number of new strange particles. Many important experiments in particle physics were performed with the synchrocyclotrons and the synchrotrons of the 50's with the pions, the muons, and the strange particles they produced. By the end of the decade the physics with these particles was being done almost exclusively with machines. It was no longer fruitful to look at the cosmic rays to study elementary particles.

It seems reasonable to suggest as Alvarez has,[14] that modern particle physics had its start in 1946, during the last days of world War II, when a group of young Italians, Conversi, Pancini, and Piccioni, while hiding from the Germans, carried out a remarkable experiment.[15] They showed that the "mesotron" which had been discovered in 1937 by Neddermeyer and Anderson[16] and by Street and Stevenson,[17] was not the particle predicted by Yukawa as the mediator of nuclear forces, but a weakly interacting particle we now call the muon. The Yukawa particle, now known as the pion, was discovered the following year by Occhialini, Powell, and collaborators.[18] This group from Bristol used a new nuclear emulsion technique developed in collaboration with Ilford Laboratories. After exposure to cosmic rays they not only found the pions but showed them decaying into muons.
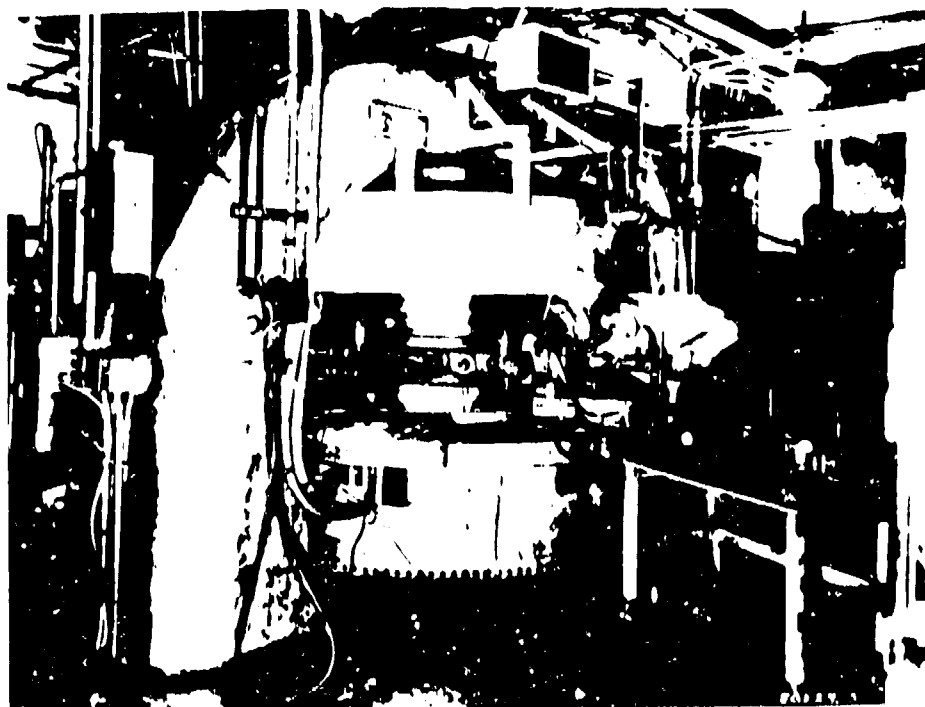
Fig. 6: The 37" cyclotron built by J. R. Dunning at Columbia University. A bombardment of uranium by neutrons is being carried out by H. L. Anderson. The year is 1939.
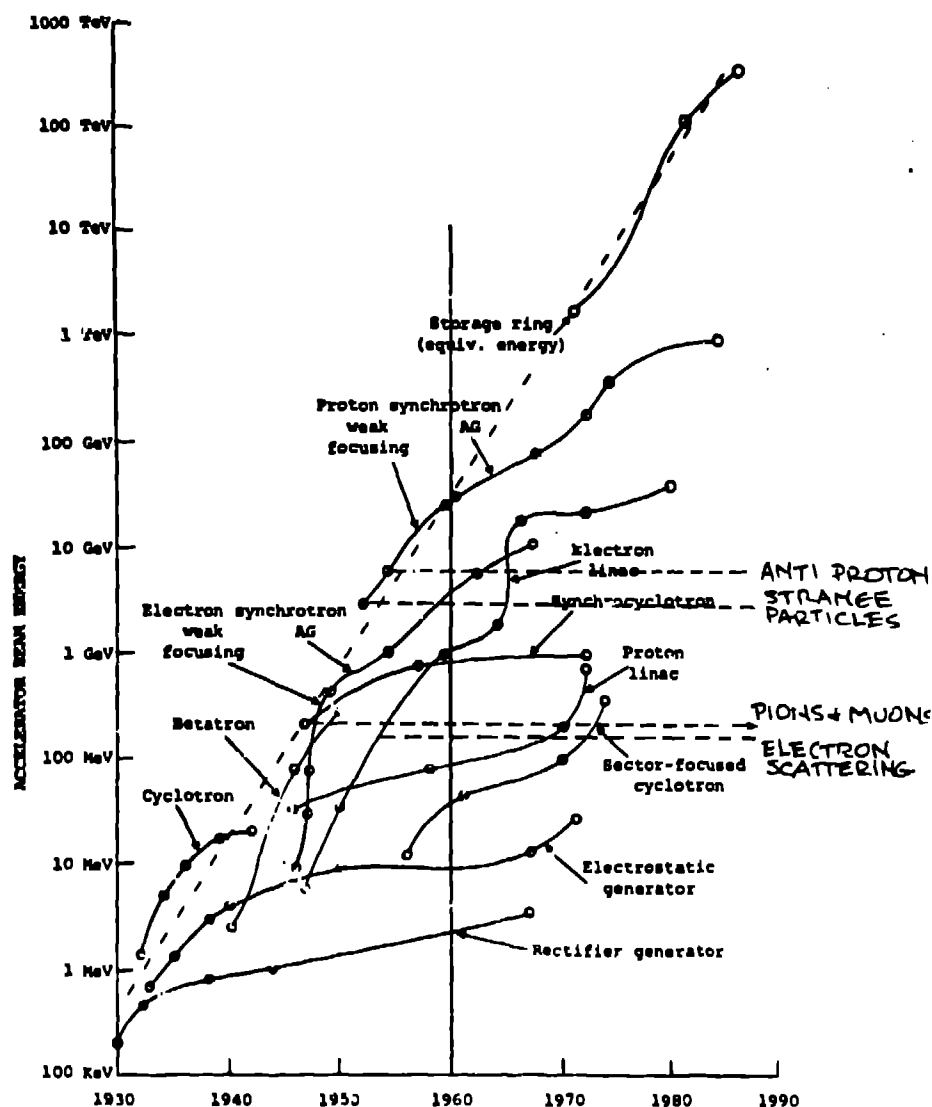
Fig. 7: Development of accelerators. The laboratory energy, or its equivalent in the the case of colliders, of particles produced by accelerators over a 50-year period.

While this was going on in Europe and England, two new great accelerators were being built in Ernest Lawrence's laboratory in Berkeley.[10] Both were based on the principle of phase stability as developed by McMillan and independently by Veksler, toward the end of the war. Lawrence's 184-in synchrocyclotron was capable of accelerating protons to an energy of 350 MeV. McMillan's electron sychrotron could reach 330 MeV. The synchrocyclotron delivered its first beam just before midnight, November 1, 1946. Although pions were being copiously produced, attempts to find them failed for lack of the proper emulsion technique. They were found almost immediately after Lattes arrived from Bristol with the technique and the proper Ilford emulsions. Lattes was the young Brazilian who, working with Occhialini and Powell at Bristol, was the first to find pions in the cosmic rays. Now he found them produced artifically in a machine. Figure 8 shows Cesare Lattes and Eugene Gardner preparing an emulsion exposure at the synchrocyclotron.

This success was soon followed by the important discovery of the neutral member of the pion family by Bjorklund, Crandall, Moyer, and York[19] at the 184-in machine. They obtained a Doppler-shifted gamma ray spectrum that could only be interpreted as arising from the decay of the π° into two gamma rays. This interpretation was confirmed soon thereafter by a more elegant experiment carried out at the 330 MeV synchrotron by Steinberger, Panofsky, and Steller.[20] They detected directly the coincidence in the emission of the two gamma rays into which the π° was expected to disintegrate. Quite independently, the π° was detected in cosmic rays at Bristol by Ekspong, Hopper, and King[21] who observed the two photon decay in emulsion and measured the lifetime as being less than 5×10^{-14} s.
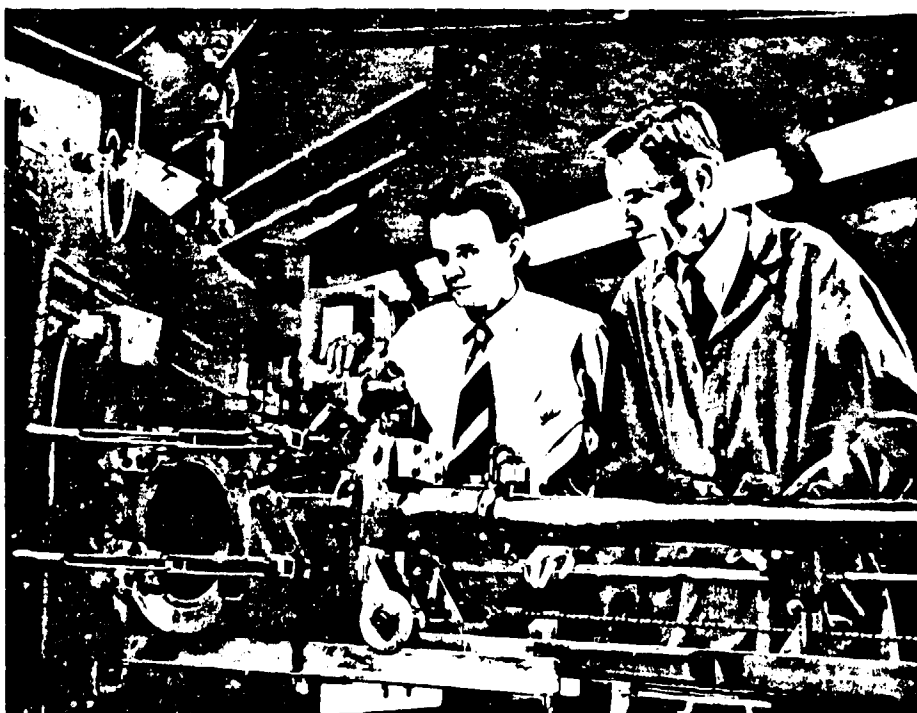
Fig. 8: Cesare Lattes and Eugene Gardner preparing an exposure of emulsions to pions in the Berkeley 184-inch synchrocyclotron.

When the cross sections for the photoproduction on hydrgen of π° were compared with those that had been made for π^+,[22,23] they were found to be about equal. Moreover, the angular distribution appeared to be isotropic in both cases. This seemed difficult to reconcile with any of the theories being discussed at the time. The first suggestion that the anomalous behavior in photoproduction might be due to the existence of a nucleon isobar was made by Fujimoto and Miyazawa[24] and also by Brueckner and Case.[25] The argument did not become convincing until after the discovery of the resonance in the pion-proton scattering. It then became possible for Brueckner and Watson[26] to put the photoproduction results on a firmer footing.

Many important experiments were done with accelerators during the 50's. Among them, I want to mention the beautiful experiments of Hofstadter[27] using the electron linac at Stanford. They gave quantitative evidence for the finite size of the proton and a glimmer of the tiny world within and the 4th spectroscopy that has preoccupied us since.

Synchrocyclotron at Chicago

Instead of reviewing these developments more completely, I thought it might be more interesting to tell about the third experiment in some detail. This was the experiment in which the pion-proton resonance appeared unexpectedly in a striking way. The work began in 1951, soon after the construction of the synchrocyclotron was completed at Chicago.[28] This machine was designed to accelerate protons to 450 MeV, 100 MeV more than its predecessor at Berkeley, so that the intensity and energy of the pion beams it could produce would be substantially greater. During the construction of the machine, I kept Fermi closely coupled to all the developments. It was understood that once the machine was completed, we would resume our work together. When the time came we organized a small group, including some graduate students, and began a series of measurements on pion scattering. John Marshall, who helped design and build the machine formed his own group. Other members of the Institute for Nuclear Studies also formed groups and used the machine according to a schedule that was worked out each week. Figure 9 is a photograph showing Enrico Fermi, myself, and John Marshall, at the cyclotron.

Fig. 9: Enrico Fermi, Herbert Anderson, and John Marshall at the Chicago synchrocyclotron.
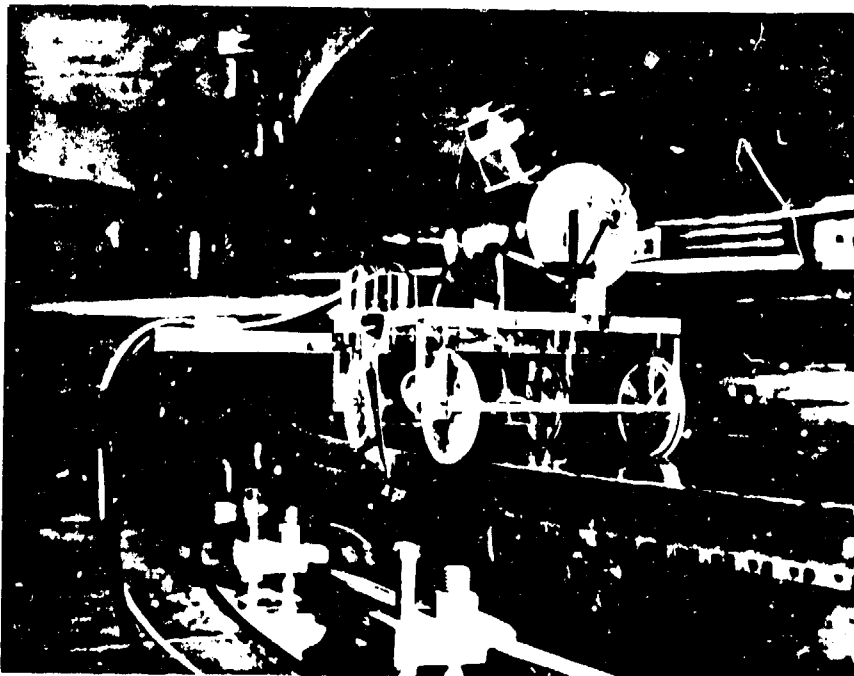
Fig. 10: The "Fermi trolley", a movable target for the proton beam inside the cyclotron capable of obtaining the beam intensity from temperature difference measurements.

When I looked among my collection of notebooks for the ones of that period, I found somewhat to my suprise, that in some sections the entries were almost entirely in Fermi's hand. It is possible to catch the excitement of discovery in those pages. They also gave an interesting glimpse of Fermi as an experimenter.

<u>Fermi Trolley</u>

Before using the cyclotron, Fermi wanted to add his contribution to its construction. He offered to take care of the target arrangements. One weekend, he went into the shop and built the trolley car shown in the photograph of Fig. 10. It was an ingenious device and became so useful it remained in operation for many years. Mounted on the edge of the magnet pole inside the vacuum, the trolley car could be moved around by manipulating a set of switches outside the vacuum chamber. Each pair of wheels was on an axle to which was attached a magnet coil. The coils were set at 90° to one another. Sending current through the coil in the horizontal position with the cyclotron magnet on would turn it to the upright position. This rotated the wheels through 90° and brought the second coil to the horizontal position. By sending current through the second coil, the wheels would rotate by an additional 90°. Switching the current from one coil to the other would send the car around the pole in one direction. Reversing the current moved the car in the op-posite direction. A third coil was used to raise or lower the target in or out of the beam.

The general scheme was to provide negative and positive pion beams at various energies as shown in Fig. 11. Fermi calculated the trajectories from a map of the cyclotron magnetic field and slots were cut in the steel shield that separated the cyclotron from the experimental room according to his prescriptions. The negative pions emitted in the forward direction came out of the cyclotron through a thin window in the vacuum chamber. Positive pions came out if they were emitted in the backward direction. The positive pion beams were of lower intensity but they came out readily when the magnetic field of the cyclotron was reversed.

On the other side of the shield, in the experimental area, a deflecting magnet was set up. It could be moved into position at any one of the slots and was used to make the final selection of pion energy. By requiring an extra bend, backgrounds from other particles coming through the slot, especially neutrons and gamma rays, were greatly reduced.
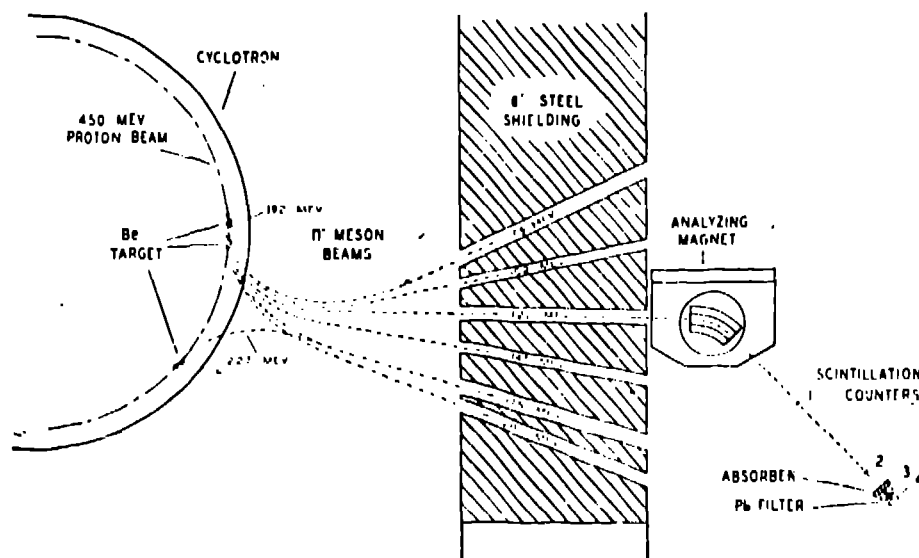
Fig. 11: Pion beams at the Chicago synchrocyclotron. Slots were cut in the steel shielding to accept pions from the target with different energies. The final energy selection was done with a dipole magnet on the experi-mental area side of the shield.
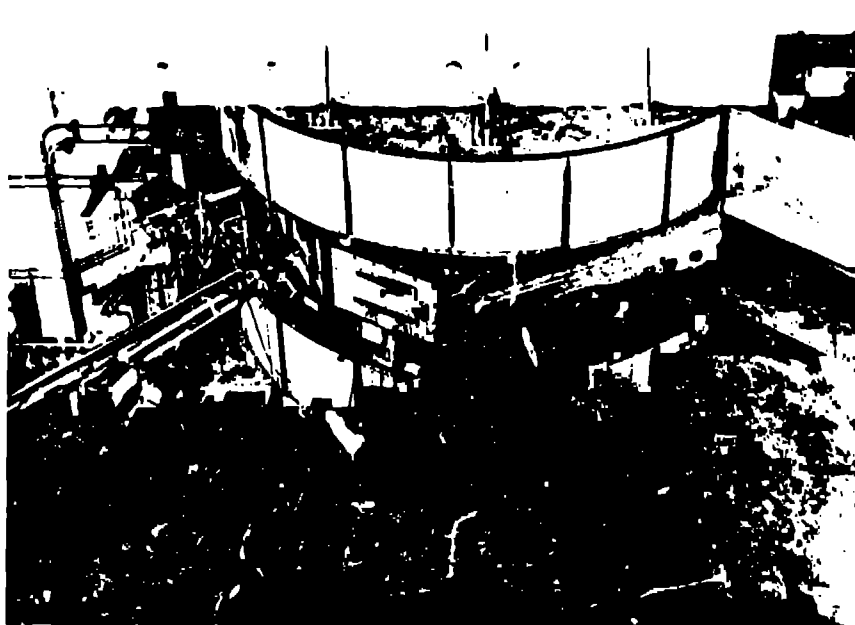
Fig. 12: Cyclotron behind its steel shield. The slot pattern cut
in steel plates for the pion beams is seen in the foreground. The
thin windows through which the pions emerged are central in the photo-
graph. A long window to the right, a short window in the center. Both
windows have their protective cover in place. Above and below the
central window are the connection terminals for the "trolley car" and
lucite windows to observe its position.

Figure 12 is a photograph of the cyclotron behind its steel shield showing the
slots in the shield. The thin window for the pions is behind its protective shield
in the long port cover to the right. There is also a thin window behind a protec-
tive shield in the smaller port cover in the center of the photograph. There are
lucite windows above and below for viewing the trolley inside. The connection ter-
minals for the trolley are mounted on these windows. The battery for energizing the
coils may be seen below the port.

The trolley car was moved to maximize the pion beam intensity. It was also used
to monitor and measure the pion beam intensity in an absolute way. This was done by
measuring the temperature of the target and determining the energy deposited by the
proton beam from a knowledge of the heat flow characteristics of the target mount.
Some of the calculations that Fermi made for this purpose are reproduced here. A
sketch of the trolley design is shown in Fig. 13. This shows the location of the
thermocouple hot junction at the target, and its cold junction at the heat sink.
Details of the heat flow calculations are given in Figs. 14 and 15. These are from
pages of one of Fermi's notebooks, dated May 25 and 28, 1951. The relaxation time
of the cylindrical heat sink is calculated on page 44, the response of the target
per microampere of beam current is given on page 45.

Detectors

When the new high energy machines, the synchrocyclotrons and the synchrotrons
of the post war period, came into operation there was an urgent need for detectors
better adapted to them. The scintillation counter arrived on the scene just in time.
They differed from the ZnS screen of the Rutherford era by being transparent to
their own radiation. Hence, they were unable in thicknesses great enough to be
sensitive to minimum ionizing particles, even gamma rays. They were made of organic

$\frac{1}{16}$" copper sheet

6"

height above base of supporting bracket $3\frac{1}{2}$"

Scale 1/2

cold junction

Chromel alumel thermocouple

hot junction

cold junction

Target

Target Copper $1\frac{1}{2}$" $\times$ .800" $\times$ $\frac{1}{4}$" full scale
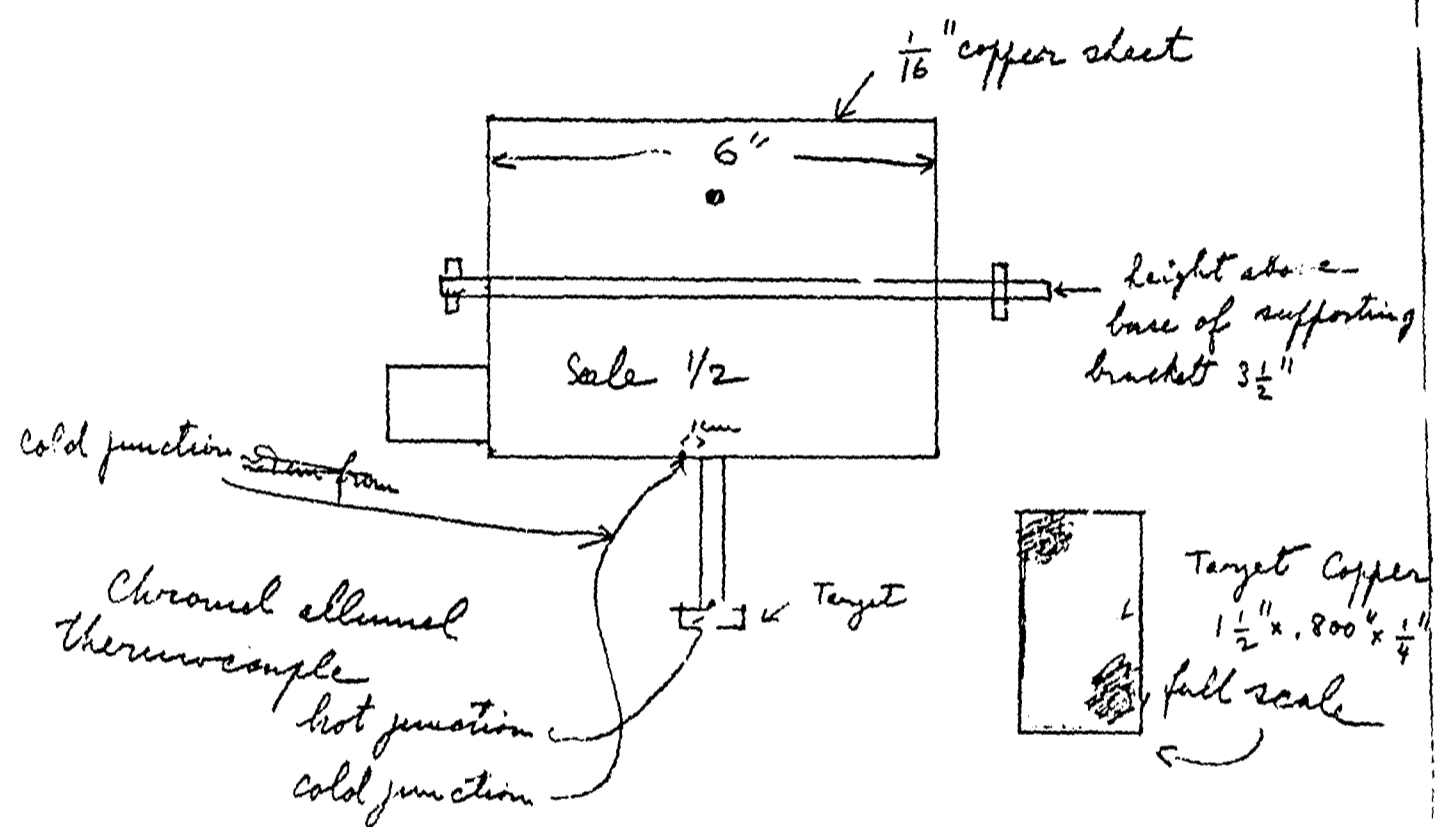
Fig. 13: Design sketch for the "Fermi Trolley" from Fermi's notebook.

Student's Name   Date 5/25/57   44

Subject Copper target for street car   Instructor's Name

coil   Target   Cylinder 788 grams

$4\frac{9}{16}$ rd   <5cm.> $4\frac{9}{16}$" od   +370 (bases + axle

$4\frac{7}{16}$" id (Heat cap. 106 cal)

6" long

Target Copper $\frac{1}{4}$" x 1.5" x .8"

Volume = .3 inch^3 = 4.92 cm^3

44.0 gr

Heat capacity = .092 x 44.0 = 4.0 cal/degree

k (Copper) = .91

Heat leak   5 cm long   wt 8 grams

area $\frac{8}{5 \times 8.9}$ = .181 cm^2 $\frac{5.0}{.18 \times .91}$ = 30.5 degrees/cal/sec

$t = \frac{4 \times 5}{.91 \times .18}$ = 120 sec   7.3° C correspond to 1 watt heat leak

Total area of cylinder 756 cm^2

" thermal capacity 106 cal/deg

Relax Time of cylinder $\dfrac{105}{.5 \times 3 \times \dfrac{6 \times 10^{-5}}{4.2 \times 10^{7}} \; 756 \times 300}$ = 2700

Sensitivity 20.5 mm per degree C (This probably due to resistance 15 + 9 ohms   heat leak from hote to cold junction

Fig. 14: Relaxation time of cylinder used as heat sink in the trolley.

Student's Name

Date 5/28/51 45

Subject Copper target for street ear   Instructor's Name

Galvanometer calibration

$$1 mm = 6.69 \times 10^{-8} amp$$
$$= 1.00 \mu volt$$

15 ohms

In air   150 mm/watt   (15 ohm galvanometer + 9 ohm thermos,

equivalent to   161 μ volts/watt

Rate of cooling in air $\tau = 130$ sec $\tau = 156$ sec
in vacuum

Computation of heat leak

$$Q = (T_1 - T_3) K \frac{a}{\ell} = \frac{2\pi K \delta}{\ln \frac{r_2}{r_3}} (T_3 - T_2)$$

$$T_1 - T_2 = \frac{Q}{K} \left\{ \frac{\ell}{a} + \frac{\ln \frac{r_2}{r_3}}{2\pi\delta} \right\}$$

$$Q = 1 watt = \frac{1}{4.18} cal \qquad K = .91$$

$$\ell = 5 \qquad a = .180$$

$$T_1 - T_2 = 7.6 \,^{\circ}C/watt$$

$$r_2 = 1.5 \qquad r_3 = .5$$

1 μ-amp = 48 watts = 365°C 144m/s $\delta = .159$

Per cm of copper: ionization energy 2.00 MeV/gr/cm²
= 17.9 MeV/cm

Cross-section (for 280 MeV events 1.19×10⁻²⁴ ; geometrical 1.00×10⁻²⁴)
For 450 MeV protons assume 1×10⁻²⁴ collisions/cm .074
Assume energy deposited per collision 80 MeV average   .074×80 = 6.0
Total energy deposited 24 MeV/cm of copper.

Fig. 15: Computation of heat leak and calculation of response to
beam intensity.

materials, originally napthalene crystals, and when connected optically to a photo-mulitplier tube they provided a pulse output of very short duration, well suited to high speed electronic counting and coincidence circuitry. The scintillator, only a few millimeters thick, could be shaped to cover a large and precisely defined area. With all these desireable properties, the scintillation counter became an instant success. The man who discovered the organic scintillation counter was Hartmut Kallmann A short report of his work appeared in the July 1947 issue of "Natur und Technik".[29,30] A complete report of Kallmann's research reached MIT, and in October Martin Deutsch made it available, in translation, to the American scientific community.[31] He also published a short note in the March 1948 issue of "Nucleonics".[32,33] Kallmann came to New York University in 1949 and soon thereafter reported his development of liquid scintillation counters,[34] extending greatly the usefulness of this technique. The Na(Tl) high Z inorganic scintillator that became so important in gamma ray spectroscopy, was discovered by Hofstadter,[35,36] who took inspiration from the report of Kallmann's success with low Z organic materials.

Pion Scattering

The scintillation counter was just what we needed for the measurement of the pion-proton cross sections. The first results were reported at the international Conference on Nuclear Physics and the Physics of Fundamental Particles, held at the University of Chicago, September 17 to 22, 1951.[37] The Conference was organized, in part, to celebrate the successful completion of the Chicago synchrocyclotron. The work had been done by Fermi, Nagle, Long, Martin, and Yodh, besides myself, but I presented the report. The arrangement shown in Fig. 16 used two 1 inch square scintillation crystals (terphenyl) to measure the incoming pions. The target was liquid hydrogen, behind which were placed two larger liquid scintillator counters to measure the number of pions remaining in the beam after traversing the hydrogen. The transmission is obtained by measuring the ratio of the quadruple to double coincidences with and without hydrogen in the target,

$$T = (Q/D)_H / (Q/D)_{NoH} .$$

This is simply related to the total cross-section σ in cm^2 through the relation,

$$T = \exp(-\sigma x) ,$$

where x is the number of nuclei per cm^2 in the target. For accurate values, corrections have to be applied for backgrounds, purity of the beam, and other effects. Six values of the cross-section were reported for π^-, one for π^+. The π^- cross-sections shown in Fig. 17, rose steeply with energy, exceeding the geometric value at 176 MeV and dropping slightly at 217 MeV. The π^+ cross-section, measured at 50 MeV was 4 times larger than the value for π^- at the same energy. However, the experimental error was quite large for the π^+ value, making the true ratio uncertain.
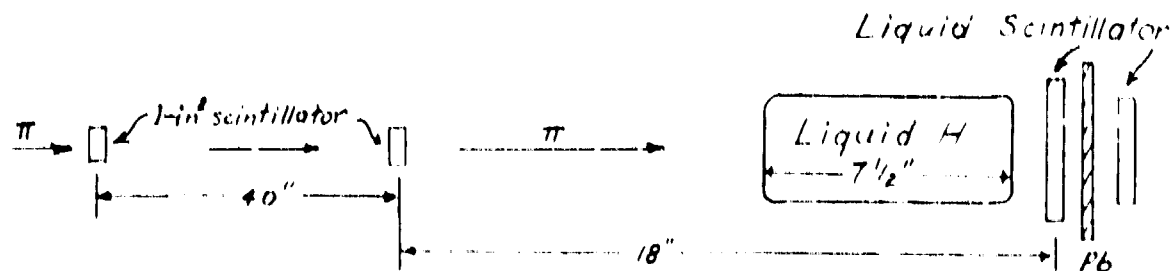

Fig. 16: Arrangement for measuring total cross sections of pions on liquid hydrogen at Chicago.

Average Energy		Cross section millibarns
50-Mev	π^+	20 ± 10
50	π^-	5 ± 15
89	π^-	29 ± 8
112	π^-	36 ± 9
135	π^-	56 ± 6
176	π^-	69 ± 7
217	π^-	60 ± 5
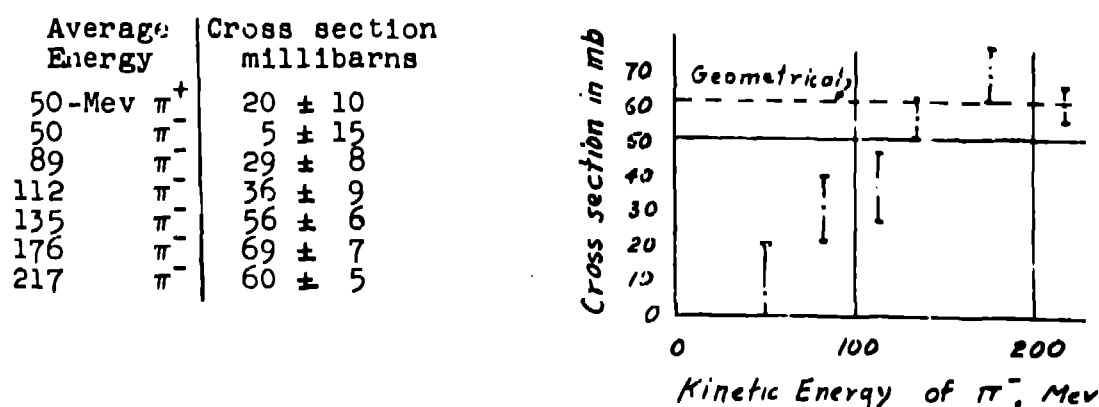
Fig. 17: Early results on total cross sections for π^- on liquid hydrogen.

Following the Conference, we went back to work determined to do everything much more carefully, especially the more difficult π^+ measurements.

Pion Beam Energy

In my notebooks of this period there were several in which Fermi had affixed his name. The title page of one of these is shown in Fig. 18. The first pages of this notebook, dated September 29, 1951, show how Fermi calibrated the deflecting magnet to measure the pion momentum. He used the stretched wire method. A current carrying wire held under tension in a magnetic field will follow the same trajectory as a charged particle with a momentum that may be deduced from the ratio of the tension to the current. Fermi measured how the trajectory shifted with wire current. The measurements begin on page 1 (Fig. 19), continuing on page 2 (Fig. 20). On page 3 (Fig. 21), the scale used in measuring the tension is calibrated. On page 4 (Fig. 22), a formula is given that relates the momentum to the weight and current. The result of the calibration is given for different target positions and for different magnet currents. The momentum is given as η in units of $m_\pi c$, the rest mass of the pion times the velocity of light.

Figure 23 shows a design of the liquid scintillation counter. This particular one came later and was used to measure the incoming pions in the angular distribution measurements. We used a prescription from Kallmann[14] for the liquid. Figure 24 is a photograph that shows me setting up the counters in the pion beam beyond the bending magnet seen in the background. Figure 25 shows Darragh Nagle working on the hydrogen target.

Returning again to the notebook, we show page 18, dated October 5, 1951 in Fig. 26. Here we see how Fermi made a careful tally of all the material in the beam to take account of the energy loss in each. This is continued on page 19 (Fig. 27), that gives the effect of multiple scattering. On page 27 (Fig. 28), we show an absorption curve in aluminum taken by Fermi in a test for proton extraction in the 122 π^- channel with all currents reversed. From the location at which he set the target, Fermi expected the proton energy to be 120 MeV and this was pretty close to what he found.

I show these samples of his work to emphasize how closely Fermi participated in the experiments. It wasn't that he felt he had to do it himself to be sure it was done right, but that he enjoyed making measurements so much that the rest of us always stood aside to let him do it.

E. Fermi

F A

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	18 Col., no desc. space
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Fig. 18: Fermi's signature on title page of one of the notebooks.

Sept 29 1951

Calibration of deflecting magnet

145 ∏ slot

34"

8' 7 ¼" on north
mini strut

26"
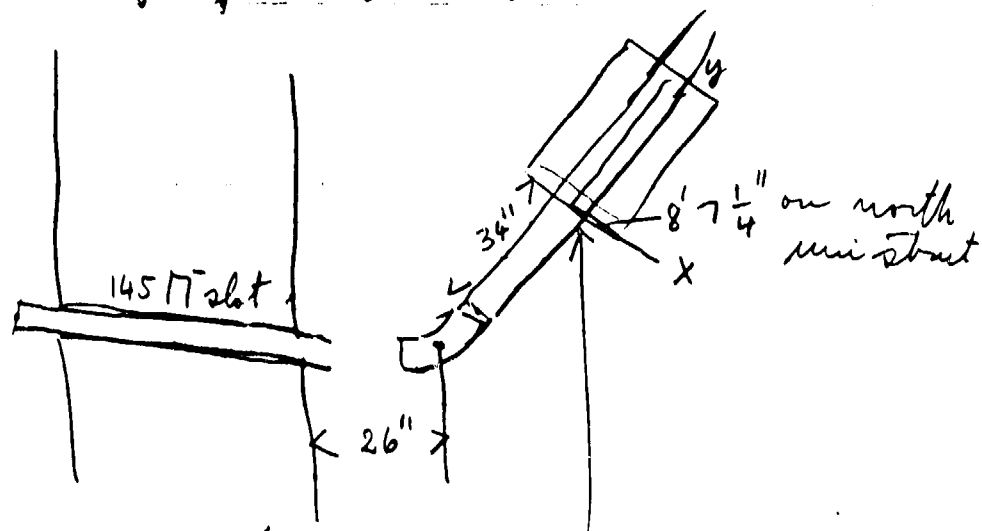
Wire measurement

Current magnet, Wt, i (wire),

Definition of trajectory x at y = 5 cm and 55 cm

I magnet	I wire	wt	x_5	x_{55}	wt/I_w
657	1751	148.7	0	0	84.9
	1182	100.6	0	0	85.1
	1183	101.5	0	1	85.8
	1183	100.0	0	-.9	84.5
	1183	100.7	-.9	0	85.1
	1183	100.7	.2	0	85.1
600	1281	102.0	.2	0	79.6
	1015	80.8	.2	0	79.6
	1015	80.5	0	0	79.6
	1483	118.5	0	0	79.9
	1481	117.2	-1	-1	79.1
	1482	116.1	-2	-2	78.3
	1480	115.0	-2	-3	77.7
	1472	118.0	0	1	80.2
	1467	117.9	1	1	80.5

wire thru center
of channel at south
end

$$\frac{wt}{I_w} = 84.84 + .48\,x_5 + .95\,x_{55}$$

$$\frac{wt}{I_w} = 79.84 - .32\,x_5 + .95\,x_{55}$$

Fig. 19: Measurement of trajectory position as a function of
wire current.

I magnet	I wire	wt	x_5	x_{55}	wt/I_w
657	1303	111.5	1	1	85.6
	1303	111.5	-.1	1	85.6
	1303	110.3	0	0	84.7
	1302	109.8	-1	-1	84.3
	1302	108.7	-2	-2	83.5
700	1247	109.1	-2	-2	87.5
	1246	110.0	-1	-1	88.3
	1246	111.1	0	0	89.2
	1246	112.0	●	●	89.9
	1246	112.0	0	1	89.9
	1246	111.0	0	0	89.1
	1246	109.5	0	-1	87.9
	1246	!11 2	0	0	89.2

$\rightarrow \quad 89.07 - .21 x_5 + 1.00 x_{55}$

Turn on cyclotron magnet

700	1239	110.0	0	0	88.8	} 89.8 ?
	1021	92.7	0	0	90.8	

Move wire 1" west at south end of channel

700	1021	91.7	0	0	89.8
	1021	91.7	-.9	⊙	89.8
	1021	91.7	.2	0	89.8
	1021	92.5	0	1.1	90.6
	1021	91.6	0	-.4	89.7

$89.86 + .05 x_5 + .64 x_{55}$

657	1021	87.5	0	-.4	85.7
	1021	88.4	0	1.1	86.6
	1021	87.5	.2	0	85.7
	1021	87.5	-.9	0	85.7

$86.07 + .31 x_5 + .53 x_{55}$

Move wire 1" east at south end of channel

	1.043	90.2	+.2	0	86.1
	~~1.047~~	~~89.6~~	~~-.9~~	~~0~~	
657	1.035	89.0	-.9	0	86.0
	1.036	88.7	0	-.5	85.6
	1.036	90.0	0	1.2	86.9

$86.31 + .28 x_5 + .63 x_{55}$

Fig. 20: Tension/current measurements continued.

3

Wire at center in back

| 657 | 1.033 | 87.3 | 0 | 0 | 84.5 | } Cyclotron magnet off |
| | 1.212 | 102.7 | 0 | 0 | 84.7 | |

657	1.205	101.7		86.4	Cyclotron magnet on
	1.011	86.7		85.8	
	1.265	106.2		84.0	85.7 ?
	1.020	88.5		86.8	+.7%
	.748	65.3		87.3	1.3%
	749	65.7		87.7	
	1.175	120.1		84.7	
	1.412	120.0		85.0	

Calibration of scale
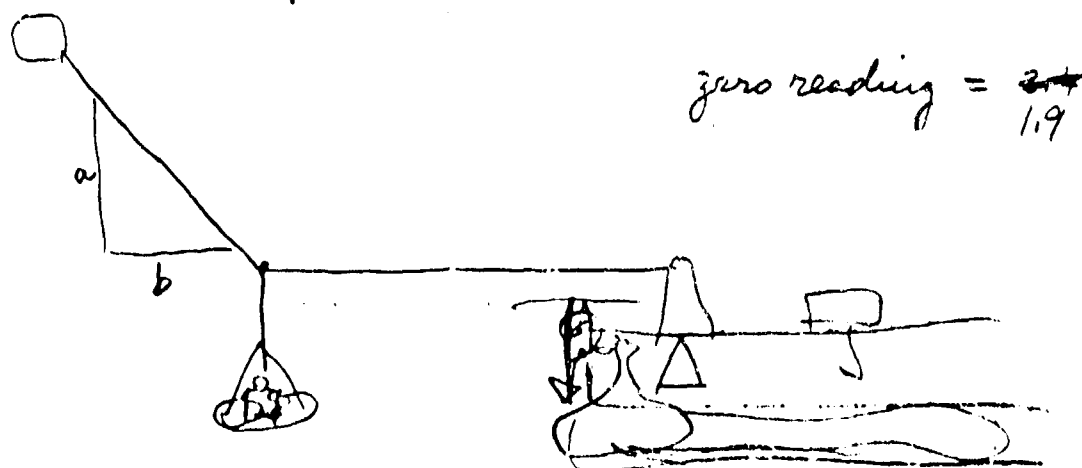
zero reading = 1.9

Wt	a	b	reading	$\frac{Wt. \times b/a}{reading - zero\ reading}$
200 gr	13.86	10.005	146.6	.998
150	13.84	10.005	110.6	.998	1.000
100	13.94	10.005	73.5	1.002
50	13.88	10.005	37.8	1.004

Fig. 21: Calibration of scale for determining tension.

4

$$HR = 10 \, g \times \frac{grams}{amps}$$

$$\eta = \frac{HR}{470,000} = \frac{980 \times 10}{470\,000} = \frac{grams/amps}{47.96}$$

$1'' \sim 1.28\%$

Position of Source	$I(magnet)$	x_5	x_{55}	η	MeV	
0	7-	0	0	1.857	156.4	
1'' west	700	0	0	1.874*	158.5	
0	657	0	0	1.769	145.5	
1'' west	657	0	0	1.795*	148.8	
1'' east	657	0	0	1.800*	149.3	
0	600	0	0	1.665	132.8	

* Cyclotron on

Note — There are magnetic parts in the scale that may have been affected

Fig. 22: Formula for the momentum from tension/current.

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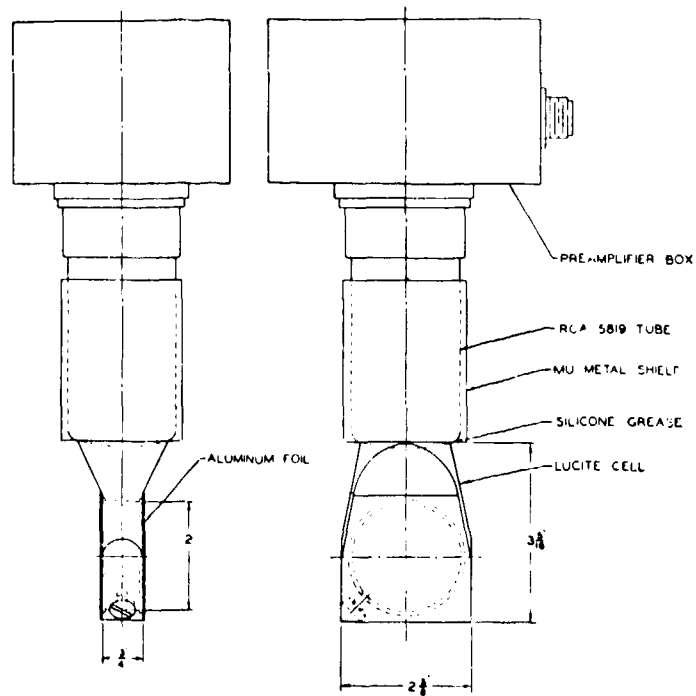
Fig. 23: Design of liquid scintillation counter.
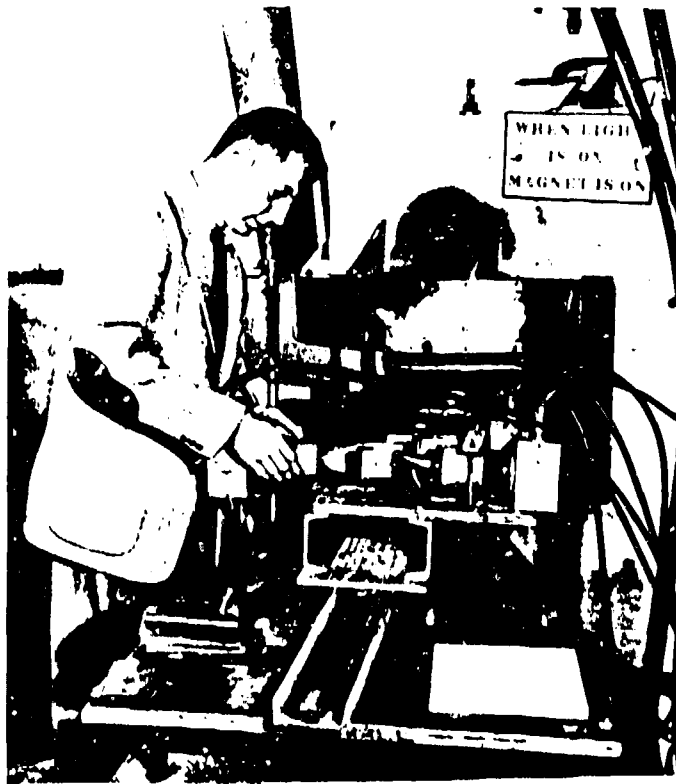
Fig. 24: Author setting up liquid scintillation counters. The bending magnet is in the background.
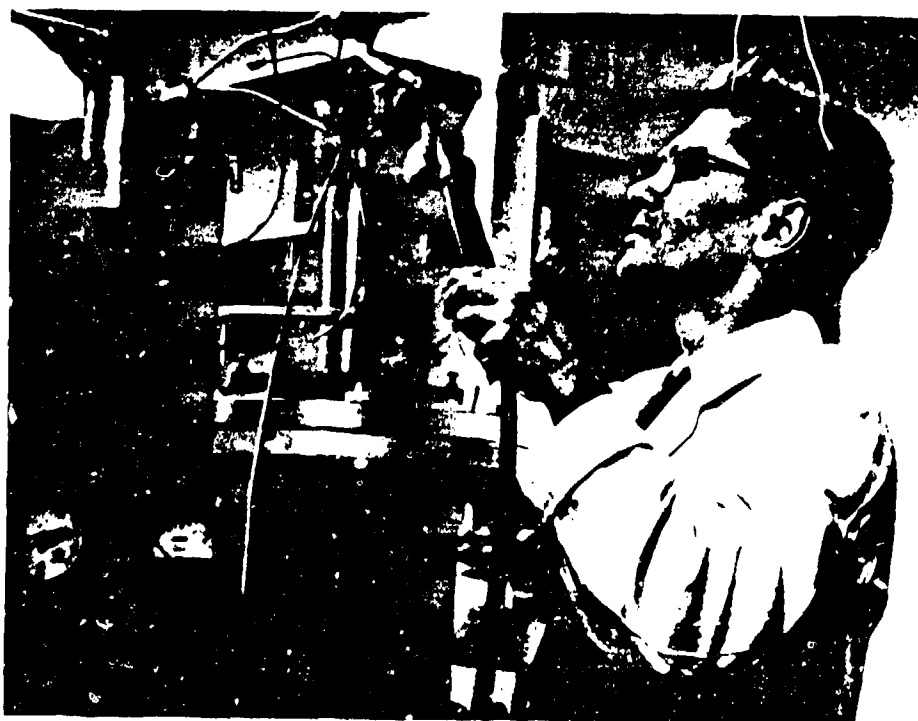
Fig. 25: Darragh Nagle working on the hydrogen target.

Cross-Section Measurements

A typical geometry for a transmission measurement is sketched, in Fermi's hand on page 16, of the notebook (Fig. 29). The date from the preceding page (not shown) was October 3, 1951. Counters 1 and 2 were 1 in^2 terphenyl crystals. Counters 3 and 4 were liquid scintillation counters. The liquid hydrogen target, 7 1/2 inches long, was inside a 10.6-inch long container and set in the space between counters 2 and 4, closer to counter 4. The basic measurement is the ratio of quadruple to double coincidences, with and without hydrogen. Gold foils were inserted when the hydrogen was removed to keep the multiple scattering of the beam the same. The liquid hydrogen was removed by pressure, its container remained in place. The 3/16-inch Pb sheet prevented proton recoils from reaching counter 3. A summary of the measurements taken with Martin "slow" circuits and not recorded in this notebook is given. The ratio of (Q/D) taken without and with hydrogen is $1.0392 \pm .0017$, and the corresponding cross-section $\sigma = (47 \pm 2) \times 10^{-27}$ cm^2.

The measurement was repeated as recorded on the next page (Fig. 30) using "Slattery fast circuits." The ratio, inverse of the transmission, was $1.0464 \pm .0033$ and the cross-section calculated from $\sigma = \ln T/x$ with $x = 8.15 \times 10^{23}$ hydrogen nuclei per cm^2 is given as $\sigma = (56 \pm 4) \times 10^{-27}$ cm^2. There were fewer accidentals with the Slattery circuits than with the Martin circuits. These measurements were done with π^- at 137 MeV. It is important to note that the effect was only 4% even though the cross-section was quite large, close to the geometric value.

Figure 31 shows page 24 of the notebook on which Fermi analyzed the data from a measurement of 175 MeV π^- on H, taken October 16, 1951. Background corrections are included explicitly, but none of the others. Again, the effect is 4.4% and the cross-section $\sigma = (54.1 \pm 3.9) \times 10^{-27}$ cm^2, about the same as at 137 MeV, so the cross-sections were leveling off.

18   Oct 5 1951

Accidental coincidences

$$\text{accid}/\text{min} = \frac{D\tau}{60} (\text{cpmin})_1 \times (\text{cpmin})_2$$

Coinciding elements	Slattery circuit	Martin circuits
#1 + #2	3×10^{-6}	
#(1,2) + #(3,4)	6×10^{-5}	$.10^{-4}$

660 amp in Magnet — Energy = 146.2 MeV thru magnet

		x/X_0	$\overline{\vartheta^2}$
Magnet	146.2 MeV		
.010" Al window — .12 MeV (.068 gr/cm²) 1.8 MeV/gr/cm²			
	146.08		
#1 crystal (1.254 gr/cm³θ, $C_{18}H_{14}$) −2.76 (2.2 MeV/gr/cm²)		.024	
	143.32		
#2 crystal	−2.76	.024	
	140.56		
.005" Al	− .06	.0013	
	140.50		
.005" Cu (.113 gr/cm²)	− .18	.0017	
	140.32		
(19 cm liq H)	−0 or −6.36	(.0097)	
	140.32 133.96	.0087	
.005 Cu	140.14 133.78	.0013	
.005 Al	140.08 133.72		

Fig. 26: Energy losses in the beam.

	$.0226x/X_0$	MeV		$\overline{\theta^2}(space)$	19
#2 crystal	5.4×10^{-4}	-142		2.40×10^{-4}	
.005" Al	.29	$\left.\begin{array}{c}\end{array}\right\}$ 140		.12	
.005" Cu	2.0		$\gamma=2$ $\eta=1.732$ $\beta=.866$		
(19 cm H)	(2.2)	~~140~~ (137.5)	$\gamma=1.915\ \eta=1.703\ \beta=.862$	0 or 1.01×10^{-4}	
.005" Cu	2.0	140 134		.88 or	
.005" Al	.29	140 134	(1.850, 1.674, .858)	.12 or	

Fig. 27: Energy loss and multiple scattering in the beam.

Maximizing beam with target position. target has been 27

cts/cm   Absorption curve of protons (Trolly at 14'0") 122 FT channel

(D.K. target × 20°
Current 18.0 millis)
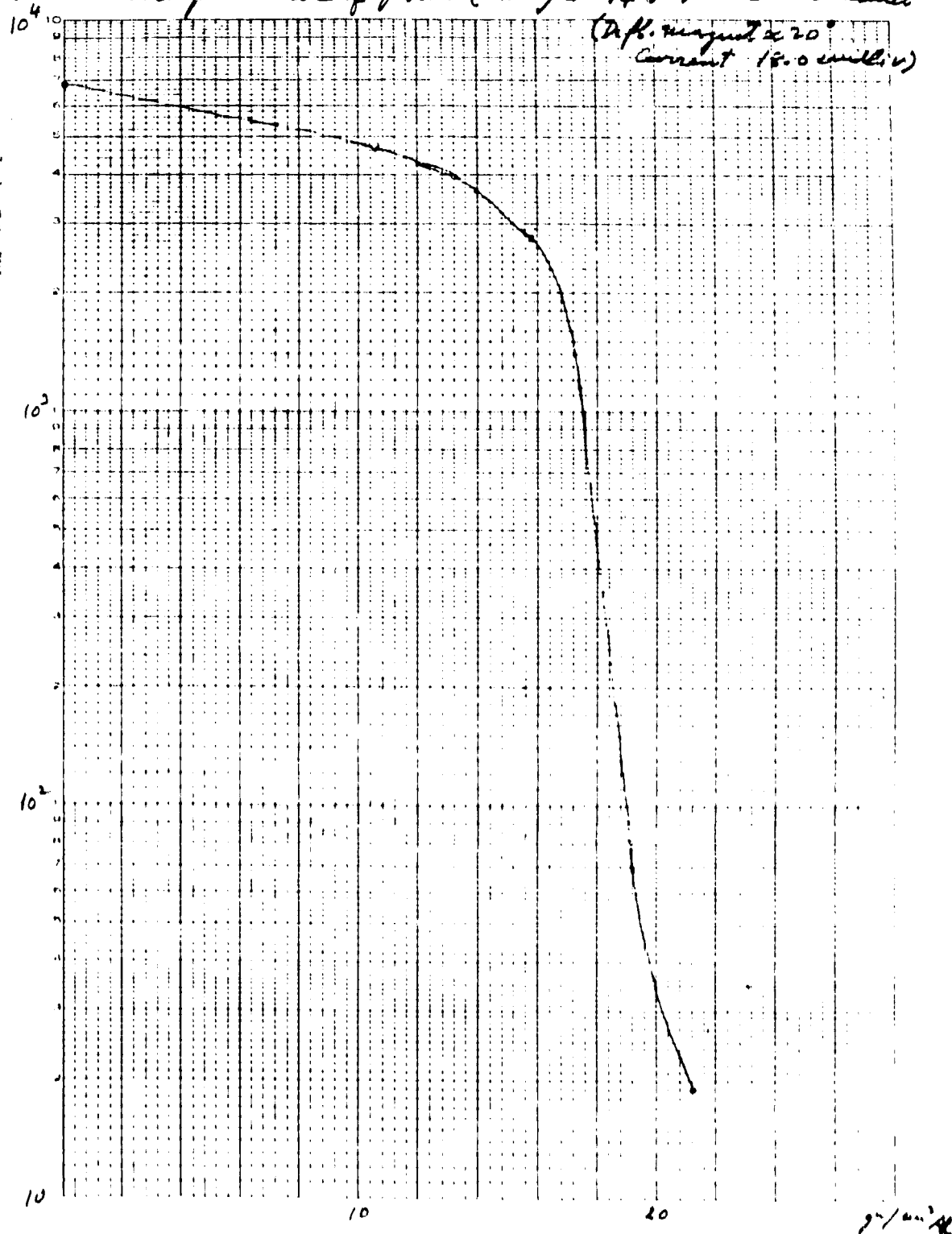
Fig. 28: Absorption curve in aluminum.

16

liq H   Au

liq H	Au		
8067	.8409		
.8065	.8520		$\frac{8383}{8060} = 1.040$
.8054	.8388		
.8051	.8354	8383	
8000	.8366		$\frac{8368}{8052} = 1.0392$
8057	.8340		
8096	8376		
8039	8346		46.
8074	8343		
8036	8384		
8124			
8024			
8052			

8060

Second geometry

①   ②   ④ $\frac{7}{16}$ Pb

36.5   106.5

111.8"   122.6"   137"

12mg Au   54mg Au

	min	wattm	m/th	D	D/m	Q	Q/m	
Au	36	352.06	9.779	160622	456.23	134416 BG	381.80 1.19	$\frac{381.80 - 1.19}{456.23} = .83425$
H	48	488.95	10.186	223977	458.08	180531 BG	369.22 1.47	$\frac{369.22 - 1.47}{458.08} = .80281$

Marlin "Slow" circuits

Corrected Ratio = 1.0392 ± .0017

$\sigma = (47 \pm 2) \, 10^{-27}$

Fig. 29: Sketch of arrangement for transmission measurements.

Doubles /watt min   <u>Fast circuits (Slattery)</u> [17]

Total minutes	Total watt min	Total Doubles	Total Quadry	D/w.m.
48	11.9 watts 571.50 Gold	123943	86909 152.07	216.87 – 2.06
48	10.8 watts 518.30 LiqH	111828	75004 144.71	215.76 – 1.88

$$\overline{\left(\frac{Q}{D}\right)} = \begin{cases} .7012 \pm .0013 \quad .7012 \pm .0016 \quad \text{Corrected for } 2.65\% \; .7064 \\ .6706 \pm .0015 \quad .6707 \pm .0017 \quad .6751 \\ \qquad\qquad\qquad\qquad\qquad\qquad 1.0464 \pm .0033 \end{cases}$$

Singles/w min   #1 2989   #2 1473

$(3,4)$ doubles $= \dfrac{11506 \; \text{coinc}}{20.43 \; \text{watt min}} = 563$

N_{rows} 8.15×10^{23}

$\sigma = \left(56 \pm 4\right) \times 10^{-27}$

$\left(E \simeq 132 \right)$ $\underline{E = 137 \, KeV}$

$D\tau = \cancel{8.60} \; 3 \times 10^{-6}$
(for faster circuit #1 & 2 doubles)

#1	#2	in indep. chans
$\dfrac{24847}{1 min \times 7.1 watts}$	$\dfrac{509885}{2 min \times 7.15 watts}$	

$(1+2) \quad \dfrac{1025}{2 min \times 9.5 watts}$

$D\tau = 3 \times 10^{-6}$

Coin of $(1,2)$ with $(3,4)$ (accidentals)

$\dfrac{(1,2)}{watt min} \quad \dfrac{10747}{,24.5} \qquad \dfrac{(3,4)}{watt min} \quad \dfrac{250560}{9.4.5}$

$(12) = 10025 \quad 23.3$
$(12,34) = 1767 \quad 223 (2 min)$

$D\tau = 87 \times 10^{-6}$ / adopt from other measurements
$\longrightarrow$ / $D\tau = 10^{-4}$ for doubles in fast

Fig. 30: Measurement of total cross section by transmission method, π^- on H at 137 MeV.

24

Measurement of: Oct 16 51

Scattering of Π^- on H

175 channel in reverse (Energy in magnet 226 MeV)

Deflecting magnet set for about 33° deflection

10 min runs (Current in defl. magnet ⟨21.75 mv⟩)

liq H in
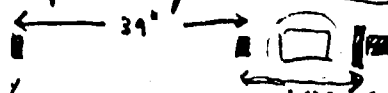
watts × min/min	D/wm	Q/wm	D/Q
87.03 / 10	73.60 − .70	61.66 − .07	1.1836
77.20 / 10	81.20 − .62	66.94 − .08	1.2052
102.47 / 10	76.34 − .82	63.57 − .09	1.1897
81.10 / 10	74.38 − .65	61.56 − .07	1.1991
99.16 / 10	67.69 − .79	55.83 − .08	1.2000
102.61 / 10	73.18 − .82	59.55 − .10	1.2172
106.09 / 10	74.11 − .85	61.25 − .09	1.1978
106.06 / 10	77.21 − .85	63.72 − .10	1.2003
122.02 / 10	83.37 − .98	69.50 − .11	1.1873
114.54 / 10	85.52 − .92	71.67 − .11	1.1822

av $1.19624 \pm .00340$

liq H out

92.34 / 10	72.95 − .74	62.96 − .06	1.1480
58.54 / 10	74.83 − .71	65.08 − .06	1.1400
86.78 / 10	77.26 − .69	67.19 − .06	1.1406
85.69 / 10	72.22 − .69	62.42 − .06	1.1470
92.10 / 10	73.54 − .74	63.29 − .07	1.1515
101.77 / 10	75.90 − .81	65.75 − .07	1.1433
98.58 / 10	77.13 − .79	66.75 − .07	1.1449
114.33 / 10	78.26 − .91	67.58 − .08	1.1459
109.19 / 10	81.82 − .87	70.45 − .08	1.1503
121.12 / 10	84.45 − .97	73.64 − .09	1.1350

av $= 1.14465 \pm .00160$

$R = \dfrac{1.19624}{1.14465} = 1.04507 \pm .0047$

$\ln R = .04406 \pm .00317$

$\sigma = \left(54.1 \pm 3.9 \right)$

Data for B.G corrections

$\dfrac{S_1}{wm} = \dfrac{1284}{1360}$

$\dfrac{S_2}{wm} = \dfrac{1377}{1357}$

$\dfrac{D_{34}}{wm} = \dfrac{446}{508}$

BG on D/wm

$\left(\dfrac{S_1}{wm} - \dfrac{D}{wm} \right)\left(\dfrac{S_2}{wm} - \dfrac{D}{wm} \right) \dfrac{3 \times 10^{-6}}{60} w$

$= (1322 - 76)(1367 - 76) \dfrac{3 \times 10^{-6}}{60} w$

$= .08 w$

BG on Q/wm $= \dfrac{10^{-6} w}{60} \left(\dfrac{D - Q}{wm} \right)\left(477 - \dfrac{Q}{wm} \right)$

Fig. 31: Π^- on H cross section at 175 MeV.

Cross Section for π^+

The π^+ measurements are given in another notebook that Fermi labeled Vol IV, December. 15, 1951--. The back page of this notebook has an index, written in Fermi's hand that is reproduced in Fig. 32. The portions that I want to present here are the 122 MeV π^+ measurements on liquid hydrogen, pages 32 to 36, and the 145 MeV π^+ measurements, also on hydrogen, pages 49 to 53. The arrangement was sketched by Fermi on page 32 on December. 21, 1951 (Fig. 33). In this case aluminum was used to compensate for the effect of multiple scattering in the liquid hydrogen and a calculation of the proper position for the hydrogen target is shown. Fermi noted the photomultiplier high voltage settings and the cable lengths. The measurements begin on page 33 (Fig. 34) and continue through page 35 (Figs. 35,36). The sequence is ABBA: $H_{in}, H_{out}, H_{out}, H_{in}$, repeated three times. The first measurement started at 11:38 AM. At 12:38 the handwriting changes from Fermi's to mine. At 13:40 it's Fermi's handwriting again until the end of the measurement at 15:16. It was clear from the first sequence of four measurements that something unusual was going on. There was a 7% effect and this was so much greater than anything we had seen before that it left Fermi shaking his head in wonder. The π^- values had been large, but they had leveled off close to the geometrical value. Here we were finding a π^+ cross-section that was substantially larger still. According to my recollection I had received, on that day, a preprint from Keith Brueckner in which he showed that the π^+/π^- ratio could be explained in terms of a nucleon isobar with spin 3/2 and isotopic spin 3/2. Fermi expressed skepticism at first. It seemed like a wild guess. But I could read from the graphs that Brueckner was predicting a cross-section of 88 mb. Our value was coming out to be 83 mb. It was pretty close, and the agreement would be even better after corrections. At this point Fermi reached for the paper and asked to be excused. He returned a short time later with a broad grin on his face. He announced, with evident satisfaction, that the cross-sections

$\mathcal{I}ndex$

Fig. 32: Index to one of Fermi's notebooks.

Dec 21 1951

3⌡ Transmission of 122 π^+ in liq H

Al compensator

$$D^2 = .108 \left(b^3 - a^3\right) \text{ (inches)}$$
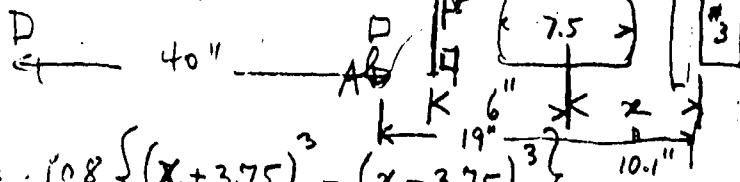
$$\left(x+6\right)^2 = .108 \left\{ \left(x+3.75\right)^3 - \left(x-3.75\right)^3 \right\}$$

$$x^2 + 12x + 36 = .108 \left(22.5\, x^2 + 105.6\right)$$

$$1.43\, x^2 - 12x - 24.6 = 0 \qquad x = \frac{6 \pm \overset{8.44}{\sqrt{36 + 35.2}}}{1.43} = 10.1$$

~~Remark of Cable from #1 (total 6 feet ... #1)~~

$V_1 = 1550$	$B_{2_1}\ 85$	$B_{2_2}\ 80$	$A_{21}\ 77$	$A_{22}\ 68$
$V_2 = 1350$				
$V_3 = 1200$	$B_{3_1}\ 79$	$B_{3_2}\ 82$		
$V_4 = 1100$	$B_{41}\ 71$	$B_{42}\ 101$		
	$A_{11}\ 75$	$A_{12}\ 82$		

Cables from Amplifier to coinc circuit

#1	11"
#2	8'
#3	24'
#4	32'

Fig. 33: Arrangement for measurement of cross section of 122 MeV π^+ on liquid hydrogen.

33.

	arm	D	Q	D/wm	Q/D
11:56 H in Al out {19.3}		0	0		
19.3	22.67	1610	1230	71.0	.714
23.7	20.53	3063	2345	70.8	.767
28.3	23.63	4760	3645	71.8	.766
32.1	24.90	6440	4952	67.4	.778
33.9	23.31	8160	6265	73.8	.763
34.9	23.85	9818	7528	69.5	.762
35.0	21.69	11403	8778	73.1	.789
	160.58 (14 min) 11.47 watts			71.0	.7698
11:58 H out Al in 25.7		0	0		
28.8	21.92	1550	1256	70.7	.810
30.2	20.49	3020	2439	71.7	.805
30.9	19.98	4400	3600	69.1	.841
31.8	20.80	5905	4848	72.3	.829
32.2	20.37	7315	6012	69.2	.826
32.5	20.42	8790	7276	72.7	.857
32.5	20.02	10240	8505	72.4	.848
	144.00 (14 min) (10.29 watts)			71.1	.8306
H out Al in 31.0		0	0		
31.4	19.88	1430	1180	71.9	.825
31.6	19.74	2870 ¹⁴⁴⁰	2375	73.0	.830
31.6	19.47	4250 ¹³⁴⁰	3516	70.9	.827
31.5	19.27	5580 ¹²³⁰	4590	69.0	.808 ±.011
30.5	17.44	6780 ¹²⁰⁰	5606	68.8	.847
30.9	19.57	8220 ¹⁴⁴⁰	6779	73.6	.815
32.1	21.39	9637 ¹⁴¹⁷	7949	66.2	.826
(transport Seg X)	136.76 (14 min) 9.77			70.5	.8248
12:38 H in Al out 11.9		0	0		
19.5	22.26	1510	1200	67.8	.794
24.6	22.03	2950 ¹⁴⁵⁰	2325	67.2	.762
28.4	22.62	4540 ¹⁵⁵⁰	3545	68.5	.788
30.5	22.21	6110 ¹⁵⁷⁰	4780	70.7	.784
32.5	22.31	7710 ¹⁶⁰⁰	6015	71.7	.772
33.5	22.77	9320 ¹⁶¹⁰	7265	70.7	.776
34.9	22.65	10943 ¹⁶⁴¹	8523	71.5	.766
	157.05 (14 min)			69.8	.7774

Fig. 34: Measurement of 122 MeV H⁺ on liquid H.

```
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Hin            29.4                                          θ/D
Al out    7?   31.4      1580, 1550 1235                     .777
12:55 PM       33.0      3440 >1550 2452                     .785
           9   34.2      4720 >1610 3665                     .780
               35.1      6330 >1610 4962                     .793
           —   35.7      7930 >1500 6165                     .754
           —   36.3      9500 >1600 7390                     .778
           —   36.4  158.52  11061  8624            70.0     .768
                  (11.31 watts)                              .776

Hout           27.0                                          
Al in          31.5      1600 >640 1300                      .813
1:14 PM        33.6      3290 >640 2720                      .840
               35.1      4930 >1660 4067                     .814
               35.9      6510 >1580 5338                     .804
               35.5      8023 >1623 6600              →.834
               35.6      9690 >1623 7957              →.815
               36.2  162.34  11313  9277                     .813
                  (11.60 watts)                     69.7     .8200

Hout           31.4                                          
Al in          36.4      1620, 1610  1360                    .840
11:28 PM       36.9      3230, 1610  2705                    .835
               37.1      4840, 1658  4020                    .817
               37.4      6490, 1610  5400                    .836
               37.5      8170 >1610  6785                    .824
               37.5      9830 >1660  8170                    .834
               37.4  161.75  11436 1606 9500                 .828
                  (11.55 watts)                     70.7     .8307

Hin      13.0  16.7               0        0                 
Al out         25.0         1770 1770    1400               .791
               30.5         3600 1830    2915               .773
               33.3         5260 1660    4108               .779
               35.0         6900 1640    5380               .776
               37.0         8620 1720    6700               .767
               37.4        10310 1690    8015               .778
               38.3  174.80 12010 1700    9351               .786
                  (12.49 watts)                     68.7     .7786
```

Fig. 35: Measurement at 122 MeV continued.

	Brown	watt min	D		Q	D/rom	Q/D	㉞ 35
H in Pl out	31.0		0		0			
	34.0		1770	1770	1360		.768	
	36.1		3560	1790	2740		.771	
	37.5		5300	1740	4080		.770	
	38.4		7100	18..	5480		.778	
	39.1		8920	1820	6890		.774	
	39.5		10660	1740	8210		.759	
	39.7	174.54	12475	1815	9644		.790	
		(12.47)				71.5	.7731	

H out AC in	28.2		0		c			
	32.0		—		—		(.834)	
	34.4		₂440		₂870		.834	
	36.4		5130	1670	4262		.824	
	37.7		6920	1790	5770		.842	
	38.4		8660	1746	7200		.822	
	38.7		10410	1750	8628		.816	
	38.4	171.45	12081	167:	9994		.817	
		(12.25 watt:)				70.5	.8272	

14:40

li out AC in	38.3		0		0			
	38.2		1690	1690	1420		.840	
	38.2		3400	1710	2640		.830	
	38.9		5090	1690	4230		.822	
	39.5		6810	1720	5650		.826	
	37.9		8540	1730	7090		.832	
	40.1		10320	1780	8570		.831	
	40.3	172.16	12096	1776	10035		.825	
		(12.30 watts)				70.3	.8296	

15:0..

H in AL out	16.8		0		0			
	25.0		1790	1790	1385		.757	
	29.7		3530	1740	2700		.756	
	33.6		—	₂770	—		(.780)	
	36.5		7300		5640		.780	
	38.4		9130	1830	7040		.765	
	39.5		11000	1870	8480		.770	
	40.0	180.78	12757	1757	9835		.771	
		(12.91)				70.6	.7709	

Fig. 36: Measurement at 122 MeV continued.

would be in the ratio 9:2:1 for the reactions in which the final states were π^+, π°, and π^-, respectively. The isotopic spin 3/2 interaction was very strong.

To see how firmly Fermi had taken hold of the idea that this might be a resonance, I looked up the notebooks he used when he worked in his office. These show that on December 24, 1951, he had written on page 91 (Fig. 37) the charge states corresponding to total isotopic spin 3/2 and 1/2 using the appropriate Clebsch-Gordon coefficients. On the next page (Fig. 38) he wrote the wave function of the incident plane wave for scattering due to a virtual $p\pi^+$ state. On page 93 (Fig. 39) is a derivation of the cross section, in the Breit-Wigner form, for scattering from such a resonant state. The next page, 94 (Fig. 40), is dated December 25, 1951. Fermi had written the expressions that take into account both the isotopic spin and the ordinary spin. The next page, 95 (Fig. 41) carries the heading, "Assuming scattering due to a single level resonance of a state I = 3/2, J = 3/2." On this page the phase shift is introduced and the theory is developed further on the next (Fig.42) and succeeding pages (not shown) to include the π^-P scattering.

Fig. 37: Pion-proton states for isotopic spin 3/2 and 1/2.

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Scattering do to virtual

P Π_+ state

P ↑

$$e^{ifz} = \sqrt{\frac{\pi}{2}} \sqrt{\frac{1}{fz}} \, i^\ell \, P_\ell^0(\vartheta) \, J_{\ell+\frac{1}{2}}(fz)$$

$\ell = 1$

$$i \sqrt{\frac{\pi}{2}} \sqrt{\frac{1}{fz}} \cos\vartheta \, J_{3/2}(fz)$$

$$\sqrt{\frac{4\pi}{3}} \, Y_0$$

$$\pi i \sqrt{\frac{2}{3}} \frac{J_{3/2}(fz)}{\sqrt{fz}} Y_0 \rightarrow \pi i \sqrt{\frac{2}{3}}$$

$$e^{ifz} \rightarrow 1 + ifz \qquad p\text{-wave}$$

$$ifz = ifz \cos\vartheta = i\sqrt{\frac{4\pi}{3}} fz \, Y_{10} = i\sqrt{\frac{4\pi}{3}} fz \, Y_0$$

Assume: matrix element with virtual state

$$\Psi_0 = \Xi g f \frac{f}{\sqrt{\Omega}}$$

Fig. 38: Incoming wave for angular momentum J = 3/2.

The entries in the notebook were interrupted after December 26, 1951 until January 3, 1952 because on December 27, we had a new run on the cyclotron and we set up to do the π^+ scattering at the next and highest energy, 145 MeV. These were recorded in the lab notebook starting on page 49 (Fig. 43) that shows the geometry used. The measurements of transmission are given on pages 50, 51, and 52 (Figs. 44, 45, and 46, respectively). We now had an unprecedented 11% effect and a cross section that continued to rise in accordance with Brueckner's predictions. After corrections, the cross section at 136 + 6 MeV turned out to be (152 + 14) x 10^{-27} cm^2, about 3 times the geometric value. The cross section was as large as it could be; Fortune was smiling at us.

All the values of the total cross section for π^+ and π^- in hydrogen were published in a series of Letters to the Editor of the Physical Review in the March 1, 1952 issue.[38,39,40] They are shown on a plot on the single page on which the positive pion results were reported. The values included the corrections for accidentals,

$$\text{Life time} \qquad b \approx e^{-\Gamma t} \qquad b^2 \approx e^{-2\Gamma t} \qquad \qquad 93$$

$$2\Gamma = 2\pi \frac{g^2 f^2}{\Omega}$$

$$\text{Units} \quad \hbar = c = \mu = 1$$

$$\oint \Gamma = \oint \pi \frac{g^2 \gamma_o^2}{\Omega} \frac{4\pi \gamma_o^2 d\gamma}{8\pi^3 \beta_o d\gamma} \Omega$$

$$\Gamma = \frac{g^2 \gamma_o^2}{2\pi \beta_o}$$

$$\beta \qquad \dot{a}_\mathbb{x} = -i g f \, b \, e^{i(\gamma - \gamma_o)t}$$

$$\dot{b} + \Gamma b = -i g f \, a_1 \, e^{i(\gamma_o - \gamma_1)t} \qquad a_1 = 1$$

$$b = - \frac{i g f \, e^{i(\gamma_o - \gamma_1)t}}{\Gamma + i(\gamma_o - \gamma_1)}$$

$$a = \frac{-g^2 f^2 \{ e^{i(\gamma - \gamma_1)t} - 1 \}}{i(\gamma - \gamma_1)\{ \Gamma + i(\gamma_o - \gamma_1) \}}$$

$$|a|^2 = \frac{g^4 f^4 \, 4 \sin^2\left(\frac{t}{2}(\gamma - \gamma_1) \right)}{(\gamma - \gamma_1)^2 \left[\Gamma^2 + (\gamma_o - \gamma_1)^2 \right]}$$

$$\oint \beta t \, \frac{4\pi}{2} t \qquad \sigma = \frac{g^4/\pi}{\Gamma^2 + (\gamma - \gamma_o)^2} \, \frac{\gamma^6}{\beta^{32}}$$

Fig. 39: Breit-Wigner formula for a virtual resonant state.

proton recoil. The plot also included the measurements made at Brookhaven and Columbia. The page is reproduced in Fig. 47.

A key statement in this paper is the one that reads, "We might point out in this connection that the experimental results obtained to date are also compatible with the more general assumption that in the energy interval in question the dominant interaction responsible for the scattering is through one or more intermediate states of isotopic spin 3/2, regardless of spin. On this assumption, one finds that the ratio for the three processes should be (9:2:1), a set of values that is compatible with the experimental observations. It is more difficult, at present to say anything specific as to the nature of the intermediate state or states. If there were one state of spin 3/2, the angular distribution for all three processes should be of the type $1 + 3 \cos^2\theta$. If the dominant effect were due to a state of spin 1/2, the angular distribution should be isotopic. If a state of higher spin or a mixture of several spin states were involved, more complicated angular distribution would be expected." It turned out that Brueckner had made the correct choice and it was the

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Dec 25 1951

$\text{State} \equiv \left(I, e, J, m \right)$

$I = \text{total isotopic spin}$

$e = \text{total electric charge} \quad \left(\text{e.g. } e = I_{\zeta} + \tfrac{1}{2} \right)$

$J = \text{total spin}$

$m = z \text{ component of spin}$

$\alpha, \beta \equiv \text{spin up or down} \qquad \Pi^{(m)} \quad \text{m or z component of orbital moment}$

$$\left(\tfrac{3}{2}, 2, \tfrac{3}{2}, \tfrac{1}{2} \right) = \sqrt{\tfrac{2}{3}}\, P_\alpha\, \Pi_+^{(0)} + \sqrt{\tfrac{1}{3}}\, P_\beta\, \Pi_+^{(1)}$$

$$\left(\tfrac{3}{2}, 0, \tfrac{3}{2}, \tfrac{1}{2} \right) = \sqrt{\tfrac{1}{3}} \left\{ \sqrt{\tfrac{2}{3}}\, P_\alpha\, \Pi_-^{(0)} + \sqrt{\tfrac{1}{3}}\, P_\beta\, \Pi_-^{(1)} \right\} +$$

$$+ \sqrt{\tfrac{2}{3}} \left\{ \sqrt{\tfrac{2}{3}}\, N_\alpha\, \Pi_\circ^{(0)} + \sqrt{\tfrac{1}{3}}\, N_\beta\, \Pi_\circ^{(1)} \right\}$$

Incident wave $\qquad P_\alpha\, \Pi_+^{\,0}$

or $\qquad P_\alpha\, \Pi_-^{\,0}$

Fig. 40: Expressions in which both isotopic spin and ordinary spin are included.

state with spin 3/2, now known as the Δ_{33}, that was dominant. However, demonstration that this was the case required measurements of the angular distribution and their analysis by the phase shift method. We quickly learned about Clebsch-Gordon coefficients and phase shift analysis, and set about doing the measurements of angular distribution, forthwith.

Figure 48, taken from the preprint Brueckner had sent me, shows the fit he obtained for the π^- cross sections. Before his paper appeared in print, Brueckner added the fit to the π^+ cross sections we had reported at the Rochester Conference on Meson Physics, held in Chicago, January 11 to 12, 1952. The overall fit shown in Fig. 49 was remarkably good. The trend of the experimental data favored the 3:1 ratio expected for a pure isotopic spin interaction even more closely than in Brueckner's calculations.

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Assuming scattering due to a single
level resonance of a state $I = \frac{3}{2}$, $J = \frac{3}{2}$
$\ell = 1$

Scattering of π^+ on P_α

Part of original wave that
is acted upon is

$$\sqrt{\frac{\pi}{2}} \sqrt{\frac{\hbar}{p^2}} \, i \, 3 \, \underbrace{Y_\ell^0}_{\cos\vartheta} \sqrt{\frac{4\pi}{3}} \, J_{3/2}\left(\frac{pr}{\hbar}\right) P_\alpha \longrightarrow$$

$$\longrightarrow \frac{\hbar}{pr} \, 3i \cos\vartheta \cos\left(\frac{pr}{\hbar} - \pi\right) =$$

$$= -\frac{3i\hbar}{p^2} \cos\vartheta \cos\frac{pr}{\hbar} = \sqrt{\frac{4\pi}{3}} \frac{3i\hbar}{2pr} \, and \, Y_1^{(0)} \left(e^{\frac{ipr}{\hbar}} + e^{-\frac{ipr}{\hbar}}\right)$$

Exact solution, however contains the term

$$\left\{\sqrt{\frac{2}{3}} \, P_\alpha \, \textit{\textbf{B}} \, Y_1^{(0)} + \sqrt{\frac{1}{3}} \, P_\beta \, Y_1^{(1)}\right\} \frac{c}{r} \, v(r) \longrightarrow$$

$$\longrightarrow \left\{ \quad \right\} \frac{c}{r} A \cos\left(\frac{pr}{\hbar} - \pi + \alpha\right) =$$

$$= \frac{cA}{2r} \left\{ \quad \right\} \left(-e^{\frac{ipr}{\hbar} + i\alpha} + e^{-\frac{ipr}{\hbar} - i\alpha}\right)$$

Fig. 41: Scattering from a single level resonance of a state
I = 3/2, J = 3/2, ℓ = 1.

Phase Shift Calculations

We carried out so many angular distribution measurements in the next six months
that Fermi began to thin that the phase shift problem might best be handled with a
computer. We had already published a short report[41] giving the phase shifts we
had calculated by hand. A more complete report was published the following
year.[42] During this period, Fermi liked to spend the summer in Los Alamos. This
time, in the Summer of 1952 he could have an electronic computer at his disposal.
The computer was the MANIAC, built at Los Alamos by Nicholas Metropolis, a close
friend, who stood ready to guide his efforts. It was typical of Fermi to learn how
the computer worked in sufficient detail to be able to operate it himself. Many of
Fermis' notes and letters of this period have been preserved by Metropolis, to whom
I'm indebted for the ones I show you now.

Figure 50 is a sample page from Fermi's Los Alamos notes. It is a description
of a program he had designated A-7-2-5, having to do with fitting the data and find-
ing the coefficients and cross sections for the angular distribution measurements.

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Dec 26 1951

$$P_\alpha \Pi_+^{\oplus} = \sqrt{\tfrac{2}{3}} \left(\tfrac{3}{2}, 2, \tfrac{3}{2}, \tfrac{1}{2}\right) - \sqrt{\tfrac{1}{3}} \left(\tfrac{3}{2}, 2, \tfrac{1}{2}, \tfrac{1}{2}\right)$$

$$P_\alpha \Pi_-^{0} = \frac{\sqrt{2}}{3} \left(\tfrac{3}{2}, 0, \tfrac{3}{2}, \tfrac{1}{2}\right) - \tfrac{1}{9} \left(\tfrac{3}{2}, 0, \tfrac{1}{2}, \tfrac{1}{2}\right) +$$

$$+ \tfrac{2}{3} \left(\tfrac{1}{2}\ 0\ \tfrac{3}{2}\ \tfrac{1}{2}\right) - \frac{\sqrt{2}}{3} \left(\tfrac{1}{2}\ 0\ \tfrac{1}{2}\ \tfrac{1}{2}\right)$$

Assume phase shift $\underline{\alpha}$ only in

$$\left(\tfrac{3}{2}, \alpha, \tfrac{3}{2}, m\right)$$

Initial state

Primary wave $\quad \hbar = 1$

$$\alpha\, e^{ipz} = \alpha \frac{\sqrt{2}\,\pi i)}{\sqrt{3}}$$

$$= P_\alpha \sqrt{6}\,\pi i \, \frac{1}{\sqrt{pz}} \, Y_1^0 \, J_{1/2}(pz) \rightarrow$$

$$\rightarrow -P_\alpha Y_1^0 \, 2i \sqrt{3\pi} \, \frac{\cos pz}{pz}$$

Scattered wave

$$\rightarrow -P_\alpha Y_1^0 \sqrt{12\pi}\, i \, a_1 \, \frac{e^{ipz}}{2pz} - P_\beta Y_1' \sqrt{12\pi}\, i \, a_2 \, \frac{e^{ipz}}{2pz}$$

Fig. 42: Scattering of π^- as well as π^+.

Figure 51 refers to the operation of this code. Figure 52 displays the phase shift formulas adapted for computer calculation, Fig. 53 is a flow diagram drawn by Fermi. Figure 54 is a sample of a program he wrote.

It became a straightforward matter to find the phase shifts that gave a good fit to the data with an electronic computer like the MANIAC. It took only five minutes once the program was in place. The trouble was that the computer found several sets of phase shifts. The phase shifts showed a plausible behavior at low energies. However, as these were followed to higher energies, among the set of phase shifts that seemed to fit the data best, the phase shift α_{33}, corresponding to the $1 = 3/2$, $J = 3/2$ state reached a maximum and turned down again without going through 90°. This was unexpected and indicated the need for further work. Fermi's reaction is shown in a letter to Metropolis, dated April 9, 1953 (Fig. 55). The problem was that the computer, given the freedom to manipulate six phase shifts without restraint, was able to find combinations that gave good fits to the data but with a non-resonant α_{33}. In the meanwhile, Hans Bethe, also a regular summer visitor to Los Alamos, interested himself in the problem and, working with deHoffman, Metropolis, and Alei[44], added plausible physical constraints that led the MANIAC to a solution that
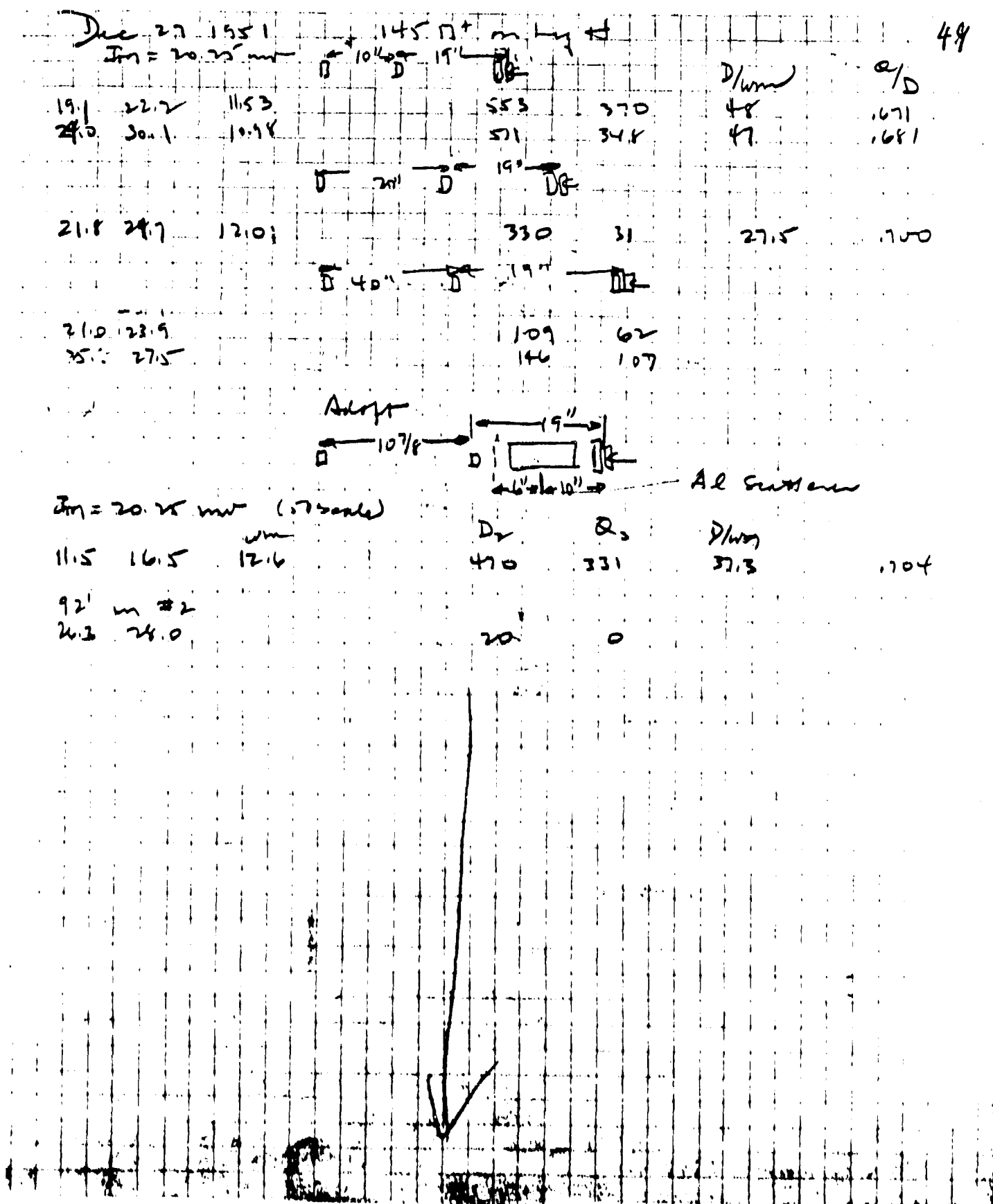
Dec 27 1951 — 145 π⁺ on Lvg H   49

$\delta_m = 20.25$ mv   D ← 10"40 → D ← 19" → DE

					D/uA	Q/D
19.1	22.2	11.53	553	370	48	.671
24.0	30.1	10.78	571	34.8	47	.681

D ← 20" → D ← 19" → DE

| 21.8 | 24.7 | 12.0 | | 330 | 31 | 27.5 | .700 |

D ← 40" → D ← 19" → DE

| 21.0 | 23.9 | | 109 | 62 | | |
| 25. | 27.5 | | 146 | 107 | | |

Adopt

D ← 107/8 → D ← 19" →   — Al scatterer

6"×6"×10"

$\delta_m = 20.25$ mv (.7 scale)

				D_r	Q_3	D/uA	
11.5	16.5	12.6		470	.331	37.3	.704

92' m #2

| 24.3 | 28.0 | | 20' | 0 | | |

Fig. 43: Arrangement for 145 MeV π⁺ on liquid H.

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Transmission of 145 17° in dig H
Geometry on back of page

				D	Q			
H in Al out	14.1	18.5	21.37	920	935	529	54T	575
H		23.5	21.22	1855	78	1.076	589	585
		24.4	20.96	2850	78	1445		
		28.9	20.63	3790		2192		
		30.5	20.54	4762		2947		5768
			105.12				45.3	
11:17 AM H out Al in	22.1	26.4	21.65	935	610			626
		29.1	21.57	1980	1000	238	626	
		31.0	21.64	2980	985	1464	601	
		36.8	20.67	39.5		2465		
		37.9	21.75	4950	3140			6343
			107.30				46.1	
11:28 AM H out Al in	33.0	33.4	21.11	1000	662	612		
		33.5	20.77	2020	1245			
		33.8	21.73	3002	2020			
		34.8	21.61	4025	2450			
		34.8	21.26	5058	3377			6676
			100.48				47.7	
11:44 H in Al out	17.7	27.3	21.90	1015	618	615		
		27.7	22.20	2055	640	1233		
		29.4	20.94	3060	1005	1810	577	
		31.4	22.04	4048	985	2410	600	
		32.7	21.90	5108	3059			.5988
			108.98				46.9	
H in Al out	33.0	33.3	20.92	1000	624			
		33.4	20.71	1975	1170			
		33.2	20.18	2975	1757			
		33.2	20.45	4047	2380			
		33.5	21.04	5073	2988		49.1	.5891
			103.30					

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			D	Q		

H out 25.8 29.8 23.75 1185 735
Al in   31.8 22.28 2318 1451
             33.0 21.95 3395 2165
             33.0 20.33 4430 2826
             33.0 20.33 5407 3489 | 49.8 | | .6453 |
                    108.64

H out 33.2 33.4 20.84 1050 683
Al in   33.4 20.57 2070 1352
             33.4 20.57 3110 2016
             33.3 20.30 4123 2711
             33.0 19.92 5077 3359 | 49.6 | | .6616 |
                    102.28

H in   17.0 22.8 21.86 1045 628
Al out   26.7 21.70 2265
             29.0 20.96 3150 1862
             30.6 21.01 4163 2465
             31.7 21.01 5255 3102 | 49.3 | | .5903 |
                    106.54

12:15
H in   28.0 30.2 21.57 1000 610
Al out   31.5 21.16 2040 1250
             32.4 20.78 3100 1855
             32.7 21.42 4040 2405
             33.1 20.93 5106 2982 | 48.1 | | 1.5840 |
                    106.26

1:06 pm
H out
Al in 26.2 24.8   1050 628
       ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
                              Scalar malfunction

       31.5 32.6 21.50 1100 785
             33.4 21.65 2245 1445
             33.8 21.36 3405 2135
             34.2 21.61 4550 2992
             34.7 8.74 5013 3278 | 52.8 | | 1.6539 |
                    94.96

1:22 H out 27.8 30.8 23.05 1173 757
      Al in   32.4 24.12 2262 1590
                   33.3 21.73 3360 2117
                   33.3 20.78 3485 2245 | 50.2 | | 1.6585 |
                    41.43

[13]

H out
52 Al in ~~~~

26.4	28.5	20.39	1068	685
	29.5	19.52	2020	1310
	30.8	19.94	3070	1980
	32.2	22.24	4100	2465
	33.7	22.78	5308	3473

104.89

50.6   .6542

H in
Al out

18.0	25.9	22.68	1100	639
	28.4	23.56	2040	1295
	31.2	22.99	3410	1937
	32.8	22.36	4517	2615
	34.0	22.58	5718	3274

114.15

50.1   .5729

Summary

H in Al out	H out Al in
.5768	6343
5988	6676
5891	6453
5903	6616
5840	6539
5729	6564
5854	6532

.116

Fig. 46: Transmission measurement at 145 MeV continued.

Total Cross Sections of Positive Pions in Hydrogen*

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(Received January 21, 1952)

IN a previous letter,[1] measurements of the total cross sections of negative pions in hydrogen were reported. In the present letter, we report on similar experiments with positive pions.

The experimental method and the equipment used in this measurement was essentially the same as that used in the case of negative pions. The main difference was in the intensity, which for the positives was much less than for the negatives, the more so the higher the energy. This is due to the fact that the positive pions which escape out of the fringing field of the cyclotron magnet are those which are emitted in the backward direction with respect to the proton beam, whereas the negative pions are those emitted in the forward direction. The difficulty of the low intensity was in part compensated by the fact that the cross section for positive pions turned out to be appreciably larger than for negative pions. The results obtained thus far are summarized in Table I.

In Fig. 1 the total cross sections of positive and negative pions are collected. It is quite apparent that the cross section of the positive particles is much larger than that of the negative particles, at least in the energy range from 80 to 150 Mev.

In this letter and in the two preceding ones,[1,2] the three processes (1) scattering of positive pions, (2) scattering of negative pions with exchange of charge, and (3) scattering of negative pions without exchange of charge have been investigated. It appears that over a rather wide range of energies, from about 80 to 150 Mev, the cross section for process (1) is the largest, for process (2) is intermediate, and for process (3) is the smallest. Furthermore, the cross sections of both positive and negative pions increase rather rapidly with the energy. Whether the cross sections level off at a high value or go through a maximum as might be expected if there should be a resonance, is impossible to determine from our present experimental evidence.

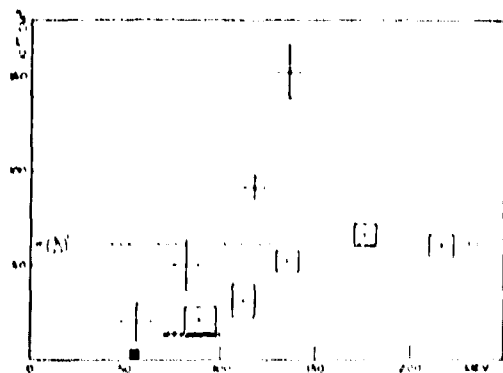Brueckner[3] has recently pointed out that the existence of a

Fig. 1. Total cross sections of negative pions in hydrogen. Codes of the triangle represent the errors and positive pions in hydrogen cross of the cross represent the errors. The cross hatched triangle is the Columbia result. The black square is the Brookhaven result and does not include the charge exchange contribution.

broad resonance level with spin 3/2 and isotopic spin 3/2 would give an approximate understanding of the ratios of the cross sections for the three processes (1), (2), and (3). We might point out in this connection that the experimental results obtained to date are also compatible with the more general assumption that in the energy interval in question the dominant interaction responsible for the scattering is through one or more intermediate states of isotopic spin 3/2, regardless of the spin. On this assumption, one finds that the ratio of the cross sections for the three processes should be (9:2:1), a set of values which is compatible with the experimental observations. It is more difficult, at present, to say anything specific as to the nature of the intermediate state or states. If there were one state of spin 3/2, the angular distribution for all three processes should be of the type $a + 3\cos^2\theta$. If the dominant effect were due to a state of spin 1/2, the angular distribution should be isotropic. If states of higher spin or a mixture of several states were involved, more complicated angular distributions would be expected. We intend to explore further the angular distribution in an attempt to decide among the various possibilities.

Besides the angular distribution, another important factor is the energy dependence. Here the theoretical expectation is that, if there is only one dominant intermediate state of spin 3/2 and isotopic spin 3/2, the total cross section of negative pions should at all points be less than $(8.3)\pi\lambda^2$. Apparently, the experimental cross section above 150 Mev is larger than this limit, which indicates that other states contribute appreciably at these energies. Naturally, if a single state were dominant, one could expect that the cross sections would go through a maximum at an energy not far from the energy of the state involved. Unfortunately, we have not been able to push our measurements to sufficiently high energies to check on this point.

Also very interesting is the behavior of the cross sections at low energies. Here the energy dependence should be approximately proportional to the 4th power of the velocity if only states of spin 1/2 and 3/2 and even parity are involved and if the pion is pseudoscalar. The experimental observations in this and other laboratories seem to be compatible with this assumption, but the cross section at low energy is so small that a precise measurement becomes difficult.

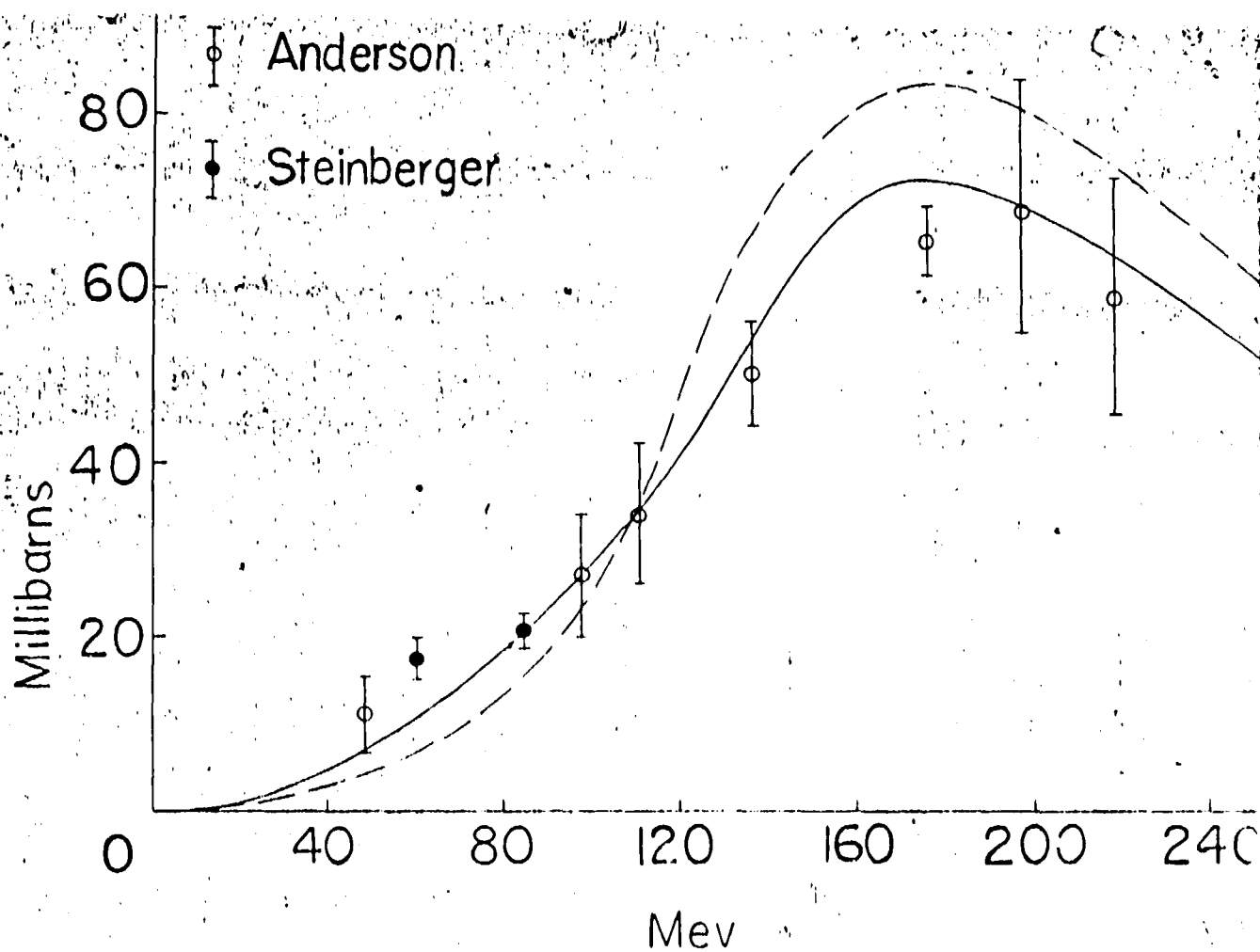| Energy (Mev)	Cross section (10⁻²⁷ cm²)
56 ± 8	20 ± 10
82 ± 7	50 ± 13
118 ± 6	91 ± 6
136 ± 6	152 ± 14

Fig. 48: Fit of cross sections for π⁻p from the Brueckner preprint received in December 1951.
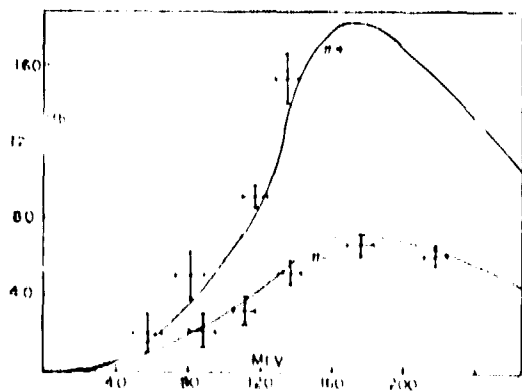

Fig. 49: Fit of π⁺ and π⁻ cross sections given in Brueckner's paper.

A - 7 - 2 - 5

Description — Contains in addition to A-7-2-4
a printing routine of the angles and conversion
to decimals and printing of coeff. and cross sections.
a routine for punching input data
An automatic operation code for coarse and
fine search of minimum and printing of results

02E - 14A Main code of A-7-2-4. Added at 032: 0400000000
for use in the subroutine computation of the angles
when /q/ located at 290 is changed. Corresponding
changes in the addresses of the orders 0FC left and
109 left.

145 - 16C Prints 013 to 018 (angles). Converts to decimals and
prints 210 to 228 (Coefficients and cross sections).

170 - 18C Automatic operation Code. Consists of 4 parts.
Part 1 (170 - 178) Is a coarse search of minimum
like in A-7-2-4 with 1° steps. The scale of Σ
is reduced to 2⁻²⁰ to avoid occasional overshooting.
When two equal successive minima are printed
automatic transfer to
Part 2 (178 - 182) Fine search of minimum. Like
above with step of 1/8 degree. Scale of Σ restored
to 2⁻¹². After two equal minima automatic
transfer to
Part 3 (182 - 187) Printing of 6 angles (hexadecimals)
9 coefficients (decimals), 7 zeros, 9 cross-sections
(decimals). On to
Part 4 — Restore main code to original form. Stop

267 - 29A Summing routine — Yields first Input + 04000000
and next 39 DD8 FF5FE

310 - 32B Conversion of input to decimals.

Fig. 50: Fermi's description of a program he wrote for the
MANIAC at Los Alamos.

A-7-2-5

Operation

Loading
                    Start   Sum

Input   600
A-7-2-5

                    02E   ~~FF~~ B03BD CACB2
                    267   40340 03885
                    310   BD223 54701

Summing routine
Start at 27E: EF000 DC199
    Finish 1   : Input + 04000 00000
    Finish 2   : 39 VD8 FF5FE

Convert to decimal
When needed Start at 317: AA268 DC312

Punch input
When needed Start at 330: EF000 FA331

Direct use of main code F
Start at 033: EF401 DC20C - Stop manually
after two equal points. Finish on bk pt 2

Direct use of printing routine
Start at 033 (or 035) to second bk pt 1
Then on to 148: AA147 DC14E ~~FF~~. If bk pt 2
is on stop after printing cross-sections. If not
goes automatically on to print minimum

Use of automatic operation
From 170: EF7FF DC034

Fig. 51: Operation of program for phase shift analysis.

$$\alpha_3 \equiv \alpha_1 \quad \alpha_1 \equiv \alpha_2 \quad \alpha_{33} \equiv \alpha_3 \quad \alpha_{31} \equiv \alpha_4 \quad \alpha_{13} \equiv \alpha_5 \quad \alpha_{11} \equiv \alpha_6 \qquad Eq\,(18)$$
$$p\,22$$

$$e_3 \equiv e^{2i\alpha_3} - 1, \quad \ldots \qquad e_{11} \equiv e^{2i\alpha_{11}} - 1$$

$$b_P = \tfrac{1}{3}(e_3 + 2e_1) \; ; \quad a_{P\beta} = \frac{\sqrt{2}}{9}(e_{33} - e_{31} + 2e_{13} - 2e_{11}) \; ; \quad a_{P\alpha} = \frac{2e_{33} + e_{31} + 4e_{13} + 2e_{11}}{9}$$

$$b_N = \frac{\sqrt{2}}{3}(e_3 - e_1) \; ; \quad a_{N\beta} = \tfrac{2}{9}(e_{33} - e_{31} - e_{13} + e_{11}) \; ; \quad a_{N\alpha} = \frac{\sqrt{2}}{9}(2e_{33} + e_{31} - 2e_{13} - e_{11})$$

$$b = e_3 \qquad ; \quad a_\beta = \frac{\sqrt{2}}{3}(e_{33} - e_{31}) \qquad ; \quad a_\alpha = \frac{2e_{33} + e_{31}}{3}$$

$$a_- = \lambda^2 \cdot \frac{2|b_P|^2 + 9|a_{P\beta}|^2}{8}, \quad b_- = \lambda^2 \cdot \tfrac{3}{2}\,\Re(b_P\, a_{P\alpha}^*) ; \quad c_- = \lambda^2 \frac{18|a_{P\alpha}|^2 - 9|a_{P\beta}|}{8}$$

$$a_0 = \lambda^2 \frac{2|b_N|^2 + 9|a_{N\beta}|^2}{8}, \quad b_0 = \lambda^2 \tfrac{3}{2}\,\Re(b_N\, a_{N\alpha}^*) ; \quad c_0 = \lambda^2 \frac{18|a_{N\alpha}|^2 - 9|a_{N\beta}|}{8}$$

$$a_+ = \lambda^2 \frac{2|b|^2 + 9|a_\beta|^2}{8}, \quad b_+ = \lambda^2 \tfrac{3}{2}\,\Re(b\, a_\alpha^*) ; \quad c_+ = \lambda^2 \frac{18|a_\alpha|^2 - 9|a_\beta|^2}{8}$$

$$\frac{d\sigma_-}{d\omega} = a_- + b_-\cos\vartheta + c_-\cos^2\vartheta$$
$$\frac{d\sigma_+}{d\omega} = a_+ + b_+\cos\vartheta + c_+\cos^2\vartheta$$
$$\frac{d\sigma_\gamma}{d\omega} = 2a_0 + 1.295\,b_0\cos\vartheta + (.778\cos^2\vartheta + .407)c_0$$

$$\lambda^2 = 11.17$$

For $\frac{d\sigma_-}{d\omega}$, Table VII angles and cross-sections marked '

" $d\sigma_+/d\omega$ Table V " " " ' ' "

$d\sigma_\gamma/d\omega$ Table VIII " " ' ' ' ' '

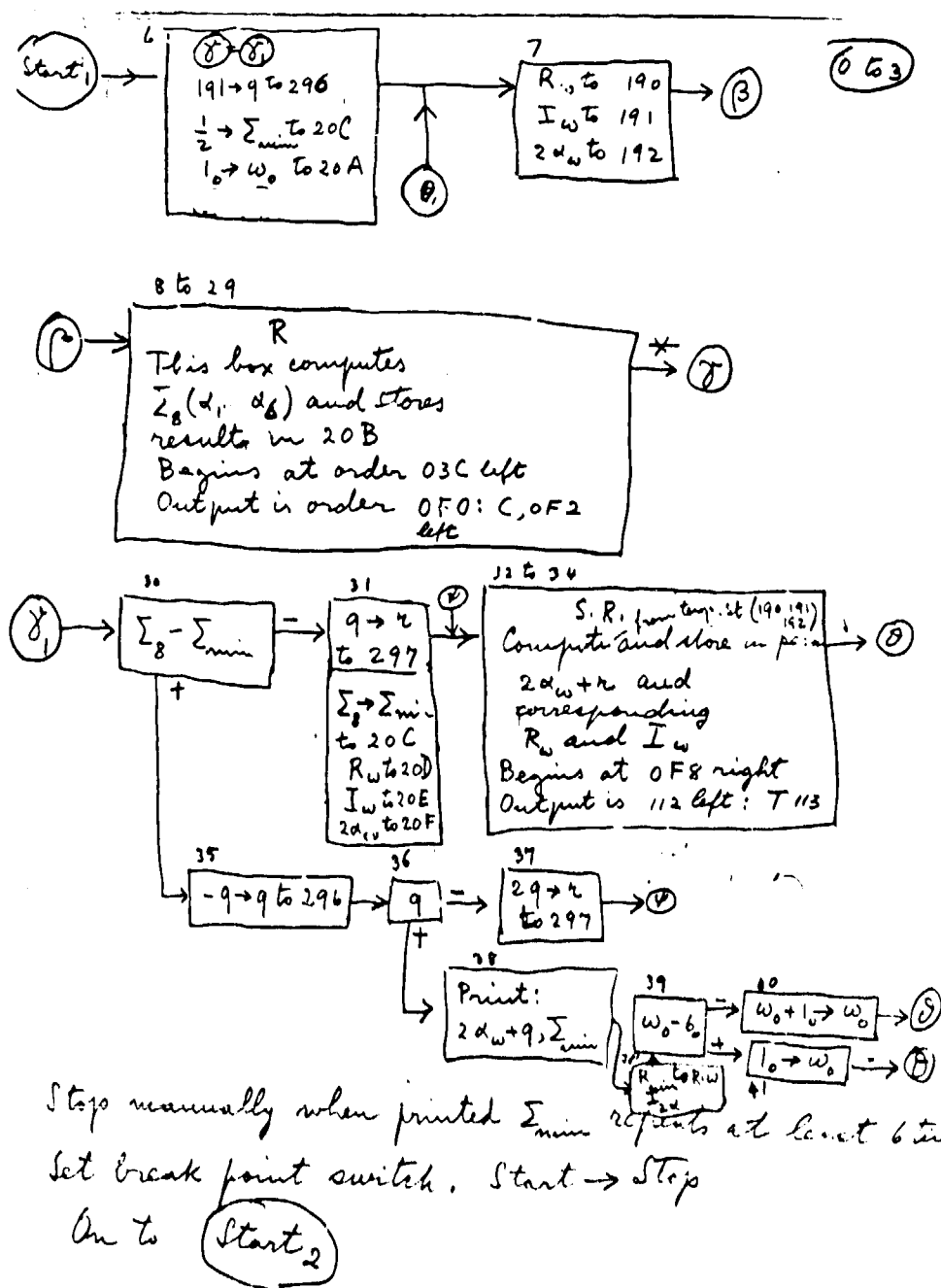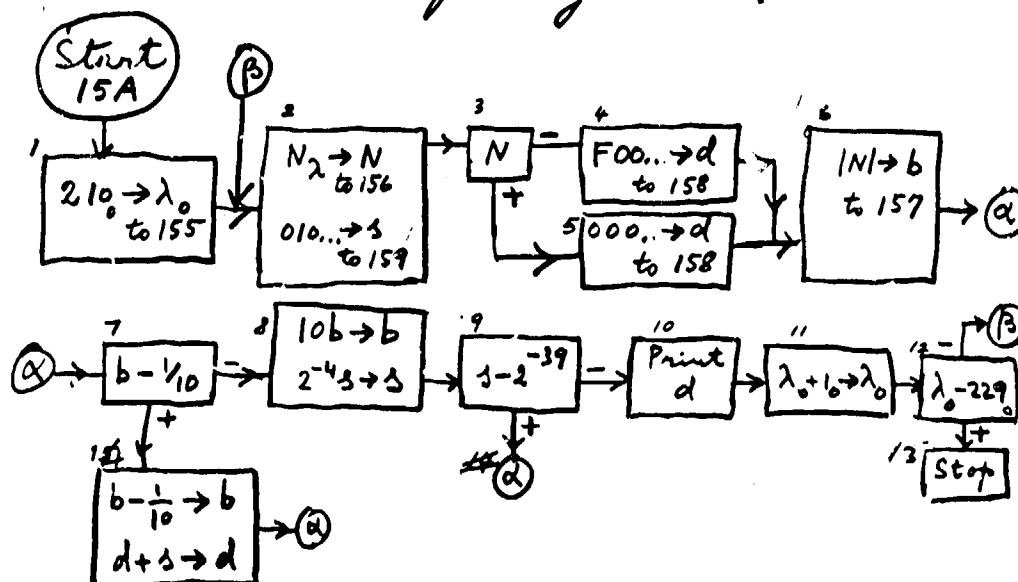Fig. 52: Phase shift formulas adapted for computer calculation.

Start₁ →

6
⟨γ⟩⟨δ⟩
191 → q to 296
½ → Σ_min to 20C
I₀ → ω₀ to 20A

θ₁

7
R_ω to 190
I_ω to 191
2α_ω to 192
→ β

0 to 3

8 to 29
C →
R
This box computes
Z̄₈(α₁, α₆) and stores
results in 20B
Begins at order 03C left
Output is order 0F0: C, 0F2
left
→ * γ

30
Σ₈ - Σ_min
+

31
q → r
to 297
Σ₈ → Σ_min
to 20C
R_ω to 20D
I_ω to 20E
2α_ω to 20F

ψ

32 to 34
S.R. from temp. st (190 191 192)
Compute and store in pair
2α_ω + r and
corresponding
R_ω and I_ω
Begins at 0F8 right
Output is 112 left: T 113
→ δ

γ₁ →

35
-q → q to 296

36
q
+

37
29 → r
to 297
→ ψ

38
Print:
2α_ω + q, Σ_min

39
ω₀ - 6₀
+

40
ω₀ + 1 → ω₀
→ δ

R_min to R·ω
2d

41
1₀ → ω₀
→ A

Stop manually when printed Σ_min repeats at least 6 times.
Set break point switch. Start → Stop
On to Start₂

Fig. 53:   Flow diagram by Fermi.

Flow diagram for converting memory 210-221
to decimals and printing results.

Fig. 54: Sample program written by Fermi.

THE UNIVERSITY OF CHICAGO

CHICAGO 37 · ILLINOIS

INSTITUTE FOR NUCLEAR STUDIES

April 9, 1953

Dr. Nicholas Metropolis
Los Alamos Scientific Laboratory
P.O. Box 1663
Los Alamos, New Mexico

Dear Nick:

Thank you for sending the results of the calculation. They are quite
interesting in that they show an energy dependence of the phase shifts
rather different from the one that had been anticipated by most. In
particular, the results do not show any evidence for a resonance in
the state 33. A resonance should be indicated by the fact that the
corresponding phase shift crosses 90o. As you may have noticed, this
is not at all the case. In fact, it appears that the phase shift in
question reaches a maximum of about 50o, and then begins to decrease.
The calculation seems to be correct in the general lines.

I have been somewhat puzzled by the fact that you noticed that for
some energies the cross sections are not obtained quite accurately,
but show some errors of a few per cent. You may remember that we had
had this trouble in other problems, and I am still somewhat uncertain
as to what is the reason for it. In any case the cross sections are
known with an accuracy worse than a few per cent, so that we should
not worry about a discrepancy of this order of magnitude.

In the attached sheets you will find the program for a new calculation
on the same experimental material. The new calculation differs from
the old one only because the smoothing out of the experimental data has
been done differently. I would like to see how much the results that
were obtained on the previous calculation depend on the way the ad-
justing is made.

Thank you again for your help.

Sincerely yours,

Enrico Fermi

EF:vr

Fig. 55: Letter, Fermi to Metropolis, about their phase shift
solutions.

up the problem as well. Using graphical methods and simplifying assumptions about the behavior of the "small" phase shifts, they were able to demonstrate that good fits were obtained with a resonance behavior for α_{33}. Fermi's comment about this work is in a letter to Metropolis dated December 22, 1953 and reproduced in Fig. 56.

Fermi didn't live to see how the whole matter was resolved. He died a year later. I like to think that the question of the 3/2, 3/2 resonance was clearly and firmly settled in a paper by U. E. Kruse, W. C. Davidon and myself[45,46] where we showed that the dispersion relations were satisfied by the resonance solution and not by the others. As new accelerators came into operation and higher energies became available, the pion-proton scattering measurements and the search for higher resonances became almost an industry. Figure 57 shows a plot of the π^+ total cross section showing the low energy portion of the data collected up to 1970.[47] The 3/2, 3/2 resonance stands out in a striking way. No wonder it would have been hard to miss. Figure 58 is a plot of both the π^+ and the π^- total cross sections on which is indicated all the nucleon isobars that had been found by phase shift analysis.[48] The I = 3/2 resonances are the Δ isobars, the I = 1/2 resonances that include neutron proton as the lowest (bound) states, are designated N. Note that because of the way interferences can plan tricks, not all the resonances are associated with a bump of the curve.

Strange Particles

While there could be some doubt about whether the Δ_{33} resonance had been established, there was no doubt that isotopic spin was an important aspect of the strong interaction. The 9:2:1 ratio had not been expected but when it showed up so dramatically in the πp scattering, it made everyone take notice. In view of this, it is suprising that the leading theorists of the time, Fermi included, were so slow to recognize that isotopic spin was the key to the puzzle of the strange particles, the major mystery of the period. The connection was made by two young theorists for whom it became a stepping stone to the fame and fortune they later came to enjoy. The first of these was Murray Gell-Mann, who will speak on this subject himself later in this Conference. The second was Kazuhiko Nishijima. Murray Gell-Mann was at Chicago at the time we were doing these experiments. He was the youngest member of our faculty. I like to think that the importance of isotopic spin, as it emerged from our experiments made a deep impression on him. The problem was to explain an apparent violation of the principle of detailed balance; how it happened that a strange particle could be made strongly, with high cross section, in a collision of a pion and a proton, and subsequently decay into a pion and a proton, but weakly, with a relatively long lifetime.

Both Gell-Mann[49] and Nakano and Nishijima[50] showed that by assigning half-integral isotopic spin to the strange K mesons and integral isotopic spin to the strange baryons, Λ° or Σ, the decay of these particles via the strong interaction would be forbidden if, in this interaction, the isotopic spin were conserved. The decay would then proceed via the weak interaction in which the isotopic spin was not conserved. Nakano and Nishijima introduced a new quantum number, later given the name "strangeness" by Gell-Mann, $S = B - 2(Q - I_3)$, where B is the baryon number, Q the charge, and I_3 the third component of the isotopic spin. Strangeness would be conserved in the strong interaction.

Not much note was taken of these ideas until 1955. At that time there was an International conference held in Pisa to celebrate the 100th anniversary of "Il Nuovo Cimento".[51] I remember the conference vividly because I was there and it made a great impression on me.

This was a conference on elementary particles, but it was devoted, almost entirely to the heavy unstable particles that were being found in the cosmic rays. In particular, there was a comprehensive report on the mesons by Amaldi who presented a compilation of every event that had been found, either in emulsions or in cloud chambers. Most of the events had been found by cosmic ray groups working

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December 22, 1953

Dr. Nicholas Metropolis
Los Alamos Scientific Laboratory
P.O. Box 1663
Los Alamos, New Mexico

Dear Nick

Thank you for your letter of December 16th with the latest news. I believe that the plan you outline is very sensible and I hope very much to hear the results. There have been, meanwhile, a number of calculations made by various people on the phase shift problems and I may give you the following set of data that have been obtained by Martin by using a graphical method and the assumption that α_{13} and α_{11} are both equal to zero. On these assumptions he claims that a solution that fits as far as they are known, both the positive and negative data is the following:

Mev	α_{13}	α_{31}
80	14°	0
101	20	0
125	32	2
150	47	5
177	72	10
190	90.5	14
205	117	48

$\alpha_3 = -10_?$

$\alpha_1 = 10°$, independent of energy.

Another attempt to fit the data with a set of phase shifts has been made here by M. Glickman. He makes even more drastic assumptions, namely, that of the p-phase shifts, only α_{33} is different from 0. On these assumptions, he obtains a reasonable fit with the following phase shifts.

Mev	α_3	α_1	α_{33}
120	-13	9	32
135	-11	11	40
169	-15	10	65
194	-9	0	81
217	-23	-4	100

All this tends to confirm that there are many acceptable solutions in the high energy region and the ultimate result probably will be that no decision is possible unless the experimental accuracy of the data is considerably improved.

There is another reason for worry at very low energy. This is due to the fact that the Steinberger data that have recently been analyzed by you give what seems to me an excessively high value for α_1. If this were the only evidence for this behavior, I would be inclined to attribute it to experimental error. On the other hand, there seems to be some supporting evidence in data from Rochester and Columbia that seem to indicate that at very low energy the cross section for elastic scattering of negative pions is much higher than I had anticipated. The situation is still very confused and probably nothing much can usefully be done about it until the experimental situation is somewhat clearer.

I hope very much to see you next January and of the two dates that you suggest, I would prefer January 22nd, because we have a number of meetings on January 21st.

With best season's greetings.

Sincerely yours,

Enrico

Enrico Fermi

EF:vr

Fig. 56: Letter, Fermi to Metropolis, about phase shift solutions by Ronald Martin and Maurice Glickman that were

mb

200

150

100

50

σ_{tot} (π^+p)

REF
20-26
22-23
38
43-70
44
47
BUBBLE CHAMBERS
COUNTERS
EMULSIONS
69

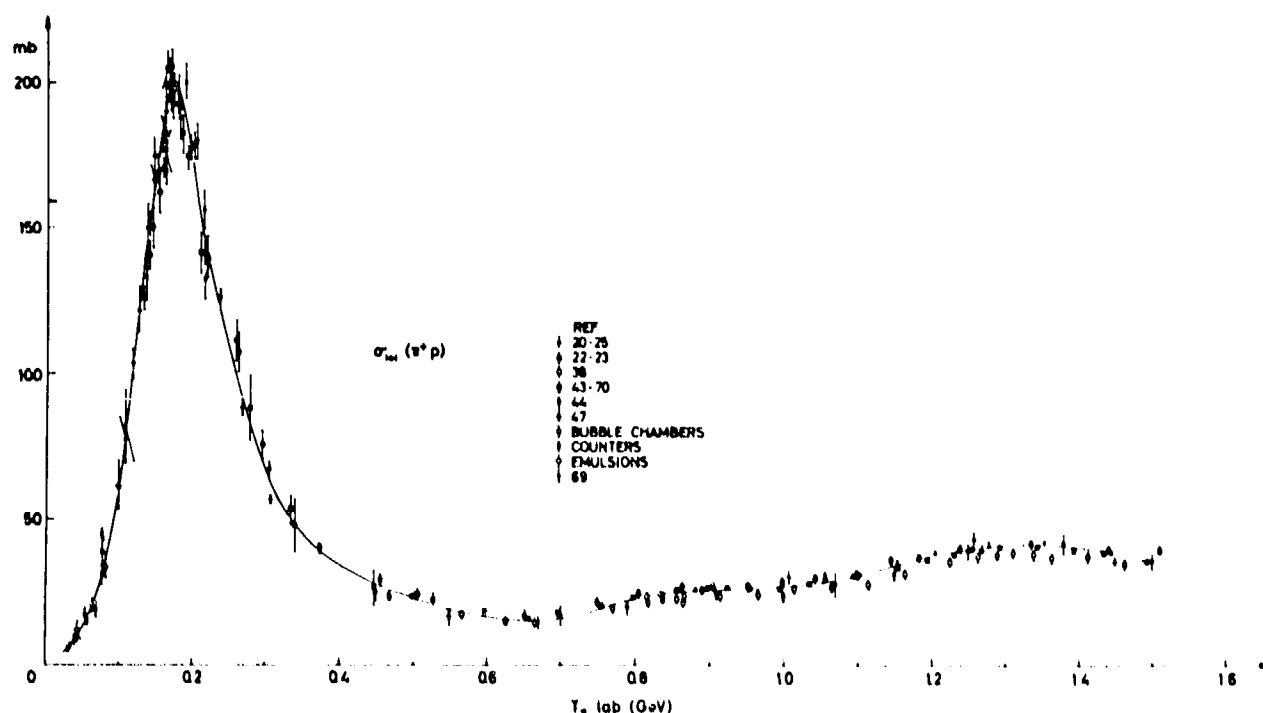0   0.2   0.4   0.6   0.8   10   12   14   16

T_π lab (GeV)

Fig. 57: Total cross section for positive pions on hydrogen up to 1.5 GeV.

of Americans working with the new accelerators in Brookhaven and in Berkeley. Accelerators were taking over in elementary particle physics in earnest. What a painstaking effort it was to collect the data on the new particles. Amaldi's collection had about 100 events obtained from a large number of contributors. There were many reports of experimental findings including one by C. F. Powell who emphasized that the mass assignments of τ^+, K^+, λ^+, Θ°, K_μ^+ were closely the same. There were many heavy unstable particles. It was hard enough to identify which particle had been found. It was harder still to determine their relation to one another. It was difficult to follow what was going on and as paper after paper was presented the confusion grew. Included in the conference was a theory session at which many of the luminaries of theoretical physics spoke, but they did not speak about the problem of the strange particles. Wigner spoke on "Relativistic Invariance in Quantum Mechanics", Heisenberg on "Hilbert Space II and the Ghost States of Pauli and Källen". Pauli spoke on "Remarks on Problems Connected with the Normalization of Quantized Fields". Many others spoke, but none had much to say about the outstanding problem in elementary particle physics and the central theme of the conference. But at the end there was Murray Gell-Mann. He had all the answers.[51] He had, by this time, put his act together. He had extended the idea of the isotopic spin to include the new heavy unstable particles and once he had made the right assignments, it worked beautifully. He interpreted the new particles as displaced charged multiplets. He called them the strange particles and introduced the idea of strangeness as a quantum number. Strangeness was conserved in the strong interaction by which they were produced and followed simple selection rules when they decayed via the weak interaction. Gell-Mann's classification of particles, strange and plain, is shown in Fig. 59. Everything fell into place in Gell-Mann's lucid and dramatic report. The spectroscopy of the elementary particles had suddenly become understandable.
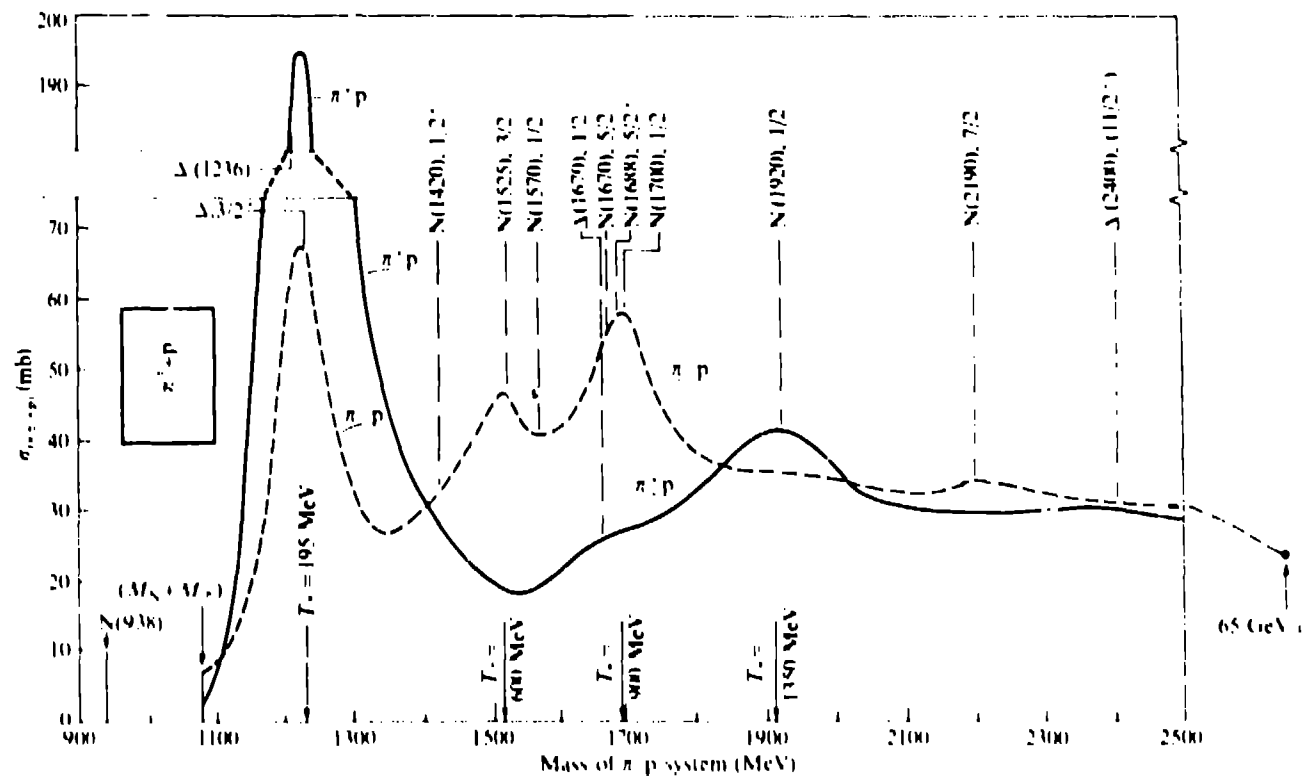
Fig. 58:   Total cross sections for π^+ and π^- to 2.5 GeV.
Isobars found by phase shift analysis are shown.

PARTICLE	ISOTOPIC SPIN	STRANGENESS	CHARGE -1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$+1$
NUCLEON	$\frac{1}{2}$	0			$n°$		p^+
ANTI-NUCLEON	$\frac{1}{2}$	0	$\bar{p}$		$\bar{n}$		
LAMBDA	0	-1			$\Lambda°$		
ANTI-LAMBDA	0	$+1$			$\bar{\Lambda}$		
SIGMA	1	-1	Σ^-		$\Sigma°$		Σ^+
ANTI-SIGMA	1	$+1$	$\bar{\Sigma}$		$\bar{\Sigma}°$		$\bar{\Sigma}$
XI	$\frac{1}{2}$	-2	Ξ^-		$\Xi°$		
ANTI-XI	$\frac{1}{2}$	$+2$			$\bar{\Xi}°$		$\bar{\Xi}$
PION	1	0	π^-		$\pi°$		π^+
K	$\frac{1}{2}$	$+1$	$\binom{K_1°}{K_2°}$		$K°$		K^+
ANTI-K	$\frac{1}{2}$	-1	$\bar{K}$		$\bar{K}$		$\binom{K_1°}{K_2°}$

Fig. 59: Gell-Mann-Nishijima scheme for classification of the strange particles.

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